M.Sc. in Geological Engineering: Subject No. 081803B02

Numerical Analysis in Geotechnical Engineering

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Lecture 6: Elasto-plastic behaviour

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Lecture 6: Elasto-plastic behaviour

6.9 Two dimensional behavior of a linear elastic perfectly plastic material
6.10 Two dimensional behavior of a linear elastic hardening plastic material
6.11 Two dimensional behavior of a linear elastic softening plastic material
6.12 Comparison with real soil behavior
6.13 Formulation of the elasto-plastic constitutive matrix
6.14 Summary

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6.1 Synopsis

- Introducing the framework and assumption of elasto-plastic material behavior
- Important concepts

6.2 Introduction

- Using the theory of plasticity to extend previous simple models with limitations
- The concepts of elasto-plastic behavior and how it is formulated for FE analysis
- Linear elastic plastic – yielding, hardening/softening, *etc*

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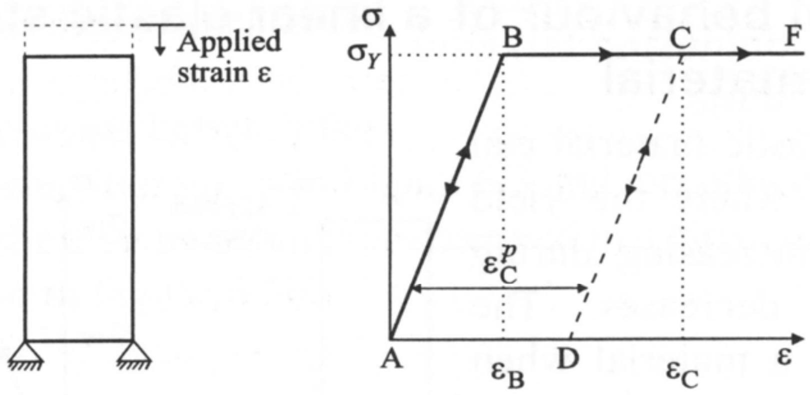
6.3 Uniaxial behavior of a linear elastic perfect plastic material

线弹性理想塑性材料



考虑最简单的加载情况——单轴压缩

Figure 6.1: Uniaxial loading of linear elastic perfectly plastic material

6.4 Uniaxial behavior of a linear elastic strain (or work) <u>hardening</u> plastic material

线弹性-塑性应变（加功）硬化材料

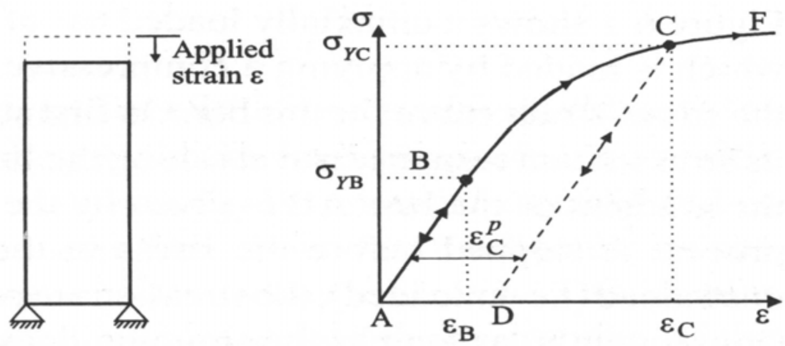


Figure 6.2: Uniaxial loading of linear elastic strain hardening plastic material

6.5 Uniaxial behavior of a linear elastic strain (or work) <u>softening</u> plastic material

线弹性-塑性应变（加功）软化材料

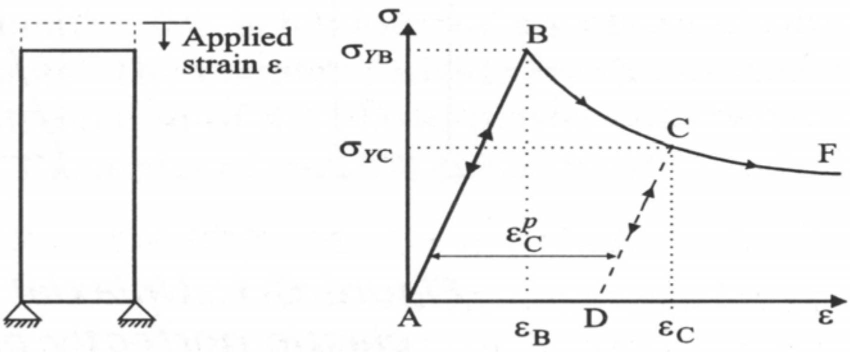
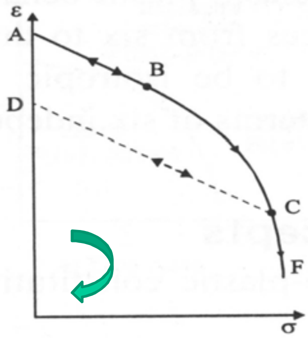


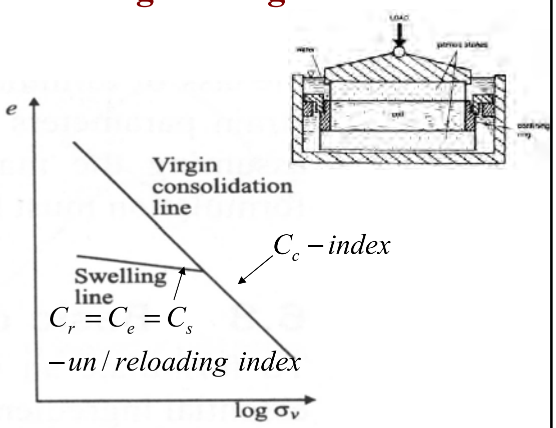
Figure 6.3: Uniaxial loading of linear elastic strain softening plastic material

6.6 Relevance to geotechnical engineering



a) Uniaxial hardening

单轴硬化



b) Oedometer test

一维固结试验

Figure 6.4: Similarity between uniaxial hardening and an oedometer test

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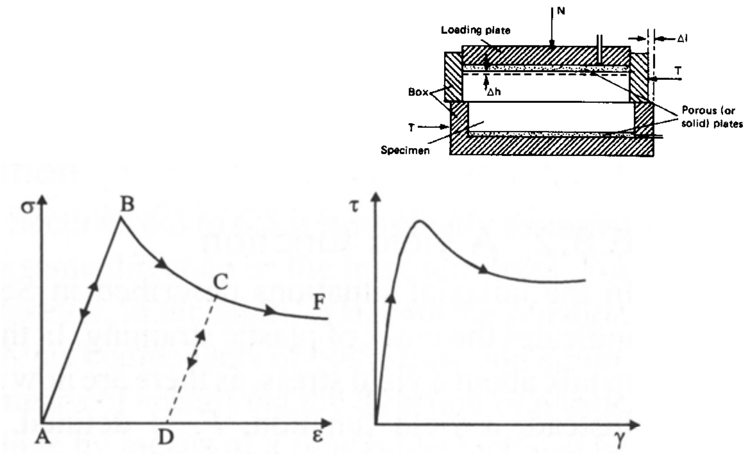

a) Uniaxial softening b) Shear box test

单轴软化 直剪试验

Figure 6.5: Similarity between uniaxial softening and direct simple shear test

6.7 Extension to general stress and strain space

广义应力和应变空间

How to extend a constitutive relation from 1-D to 3-D stress state?

We need assumptions and a mathematical model !

6.8 Basic concepts (of plasticity)

6.8.1 Coincidence of axes:

Plastic models - the principal directions of accumulated stress and *incremental* plastic strain are assumed to coincide (different from elastic model) – see sections 6.8.2 and 6.8.3

Elastic models - the principal directions of incremental stress and *incremental* strain coincide or total stress and *total* strain coincide

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Elastic behavior - General Hook's Law

$$\begin{Bmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\sigma_z \\ \Delta\tau_{xz} \\ \Delta\tau_{yz} \\ \Delta\tau_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ D_{31} & D_{32} & D_{33} & D_{34} & D_{35} & D_{36} \\ D_{41} & D_{42} & D_{43} & D_{44} & D_{45} & D_{46} \\ D_{51} & D_{52} & D_{53} & D_{54} & D_{55} & D_{56} \\ D_{61} & D_{62} & D_{63} & D_{64} & D_{65} & D_{66} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\varepsilon_z \\ \Delta\gamma_{xz} \\ \Delta\gamma_{yz} \\ \Delta\gamma_{xy} \end{Bmatrix} \qquad (5.7)$$

$$\Delta\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial \Delta u_i}{\partial x_j} + \frac{\partial \Delta u_j}{\partial x_i}\right) \quad i = 1,2,3;\ j = 1,2,3$$

$$\Delta\gamma_{ij} = 2\Delta\varepsilon_{ij} \quad (i \neq j)$$

Using increments. If D_{ij} are constants, then no difference (5.7) has 36 constants. But thermo-dynamic strain energy considerations lead [D] symmetrical – only 21 constants

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6.8.2 A yield function (*F*)

屈服函数：用来判断是否发生塑性变形

$$F(\{\sigma\},\{k\})=0 \qquad (6.1)$$

There are two types:

$\{k\}$ is dependent on plastic strain ε^p – a strain hardening/ softening parameter (vector) 塑性应变相关-应变硬/软化

$\{k\}$ is dependent on plastic work W^p – a work hardening/ softening parameter (vector) 塑性功相关-加功硬/软化

$$W^p=\int\{\sigma\}^T\{\Delta\varepsilon^p\}$$

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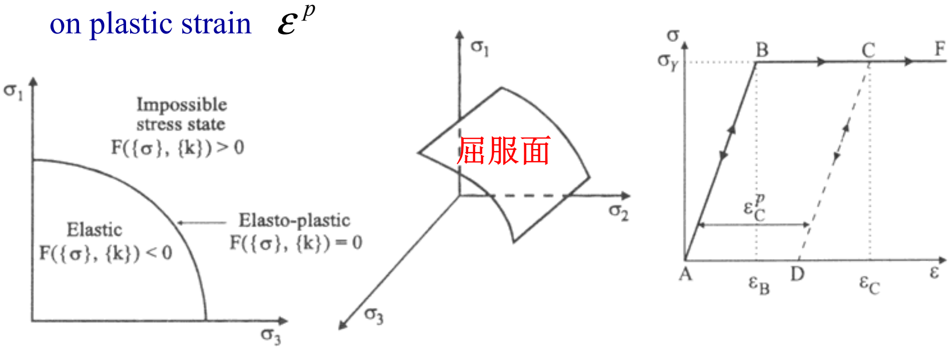
$$F(\{\sigma\},\{k\})=0 \qquad (6.1)$$

$\{k\}$ – a strain hardening/softening parameter (vector), dependent on plastic strain ε^p



a) Yield curve 二维应力状态

b) Segment of yield surface 三维应力状态

Figure 6.6: Yield function presentation

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6.8.3 A plastic potential function (*P*)

塑性势函数：用来确定塑性应变增量的方向（各分量的大小比例）

$$\Delta\varepsilon_i^p = \Lambda \frac{\partial P(\{\sigma\},\{m\})}{\partial \sigma_i} \qquad (6.2)$$

Λ – a scalar multiplier; 标量因子

P – plastic potential function;

$\{m\}$ – location of P surface (a vector), not in the final equation

$$P(\{\sigma\},\{m\}) = 0 \qquad (6.3)$$

Associated Flow Rule (Normality Condition):

$$F(\{\sigma\},\{k\}) = P(\{\sigma\},\{m\})$$

相关联、不相关联流动法则

Non-associated Flow Rule:

$$F(\{\sigma\},\{k\}) \neq P(\{\sigma\},\{m\})$$

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Principal directions of 总应力和塑性应变增量方向一致的假设 ***accumulated* (total) stress and *incremental* plastic strain are assumed to coincide**

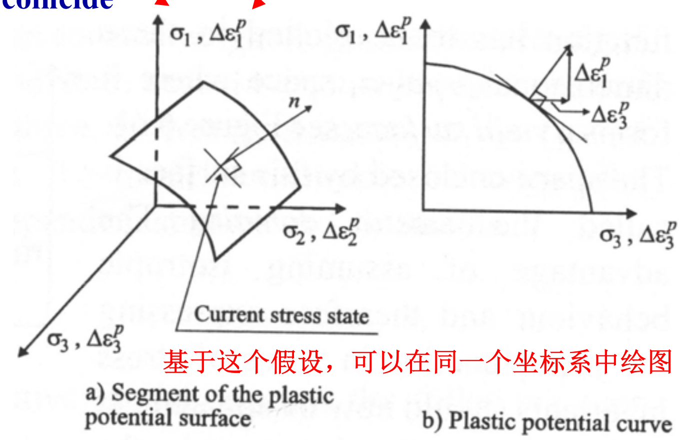

Figure 6.7: Plastic potential presentation

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6.8.4 The hardening/softening rules

硬化/软化准则：用来确定状态参数随塑性应变（功）的变化规律

$$\Delta\varepsilon_i^p = \Lambda \frac{\partial P(\{\sigma\},\{m\})}{\partial \sigma_i}$$

If a material is perfectly plastic, then no hardening/softening (k=constant); Λ will be undefined indicating the fact of infinite plastic strain at yielding

$k \neq$constant: hardening/softening – finite plastic strain ε^p

Fig.6.8 shows how yield stress varies with plastic strain for 1-D case

3-D case – yield surface varies with k – ***calibrated by 1-D test results***

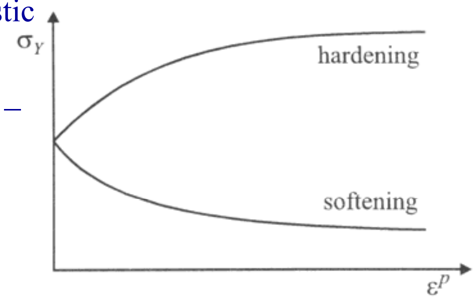

Figure 6.8: Examples of hardening/softening rules

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k – a hardening/softening parameter

$$e = e_1 - C_c \log\sigma_v$$

$$\varepsilon_v = \varepsilon_1 + \lambda \ln\sigma_v$$

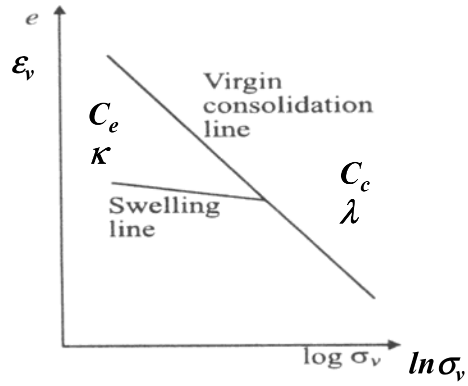
$$e = e_1^e - C_e \log\sigma_v$$

$$\varepsilon_v = \varepsilon_1^e + \kappa \ln\sigma_v$$

$$\varepsilon_v = \frac{e_1 - e}{1 + e_o}$$

$$\lambda = \frac{C_c}{1+e_o}\frac{1}{\ln(10)}$$

$$\kappa = \frac{C_e}{1+e_o}\frac{1}{\ln(10)}$$



Modified Cam-Clay model: hardening is controlled by plastic volume strain from isotropic consolidation test

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In general, having accepted coincidence of principal directions of accumulated stress and incremental plastic strain (基本假定), three further pieces of information are required to formulate an elasto-plastic model.

(a)A **yield function（屈服函数）** which signals when the material becomes plastic, and

(b)a **plastic potential function （塑性势函数）** which determines the direction of plastic straining, are compulsory ingredients.

(c)If the material hardens or softens, a **hardening/softening rule （硬/软化准则）** is required.

6.9 Two dimensional behavior of a linear elastic perfectly plastic material

Elastic below yield surface

- **Plastic strain infinite flow on the yield surface (fixed in position), however, the ratio/direction is fixed**
- **Plastic strain is finite if confined, infinite if unconfined – see footing**

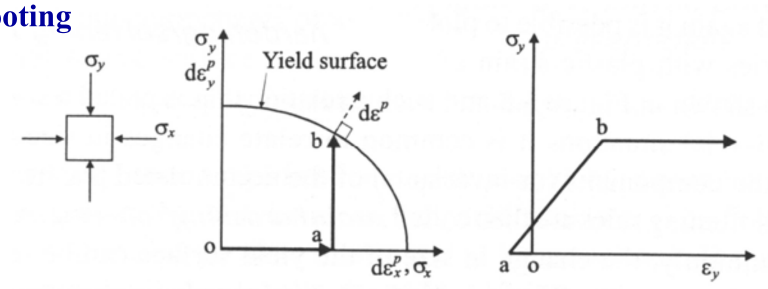


Figure 6.9: Two dimensional behaviour of a linear elastic perfectly plastic material

6.10 Two dimensional behavior of a linear elastic hardening plastic material

- Plastic strain finite flow on the yield surface (expanding or moving in position)
- Isotropic hardening – yield surface expanding
- Kinematic hardening – yield surface shifting – no change in size

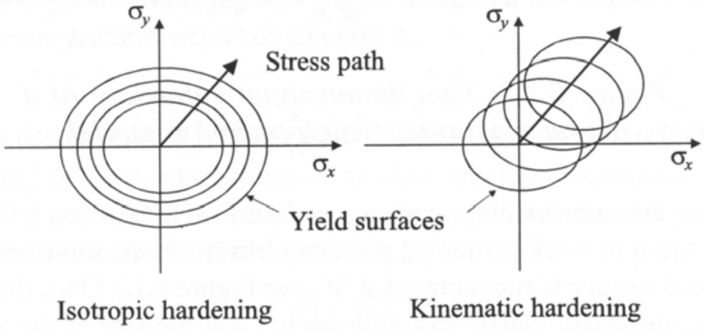


Figure 6.10: Types of hardening

- Isotropic hardening – yield surface expanding
- Kinematic hardening – yield surface shifting – no change in size
- Unloading – elastic; re-loading to *d* – plastic strain again

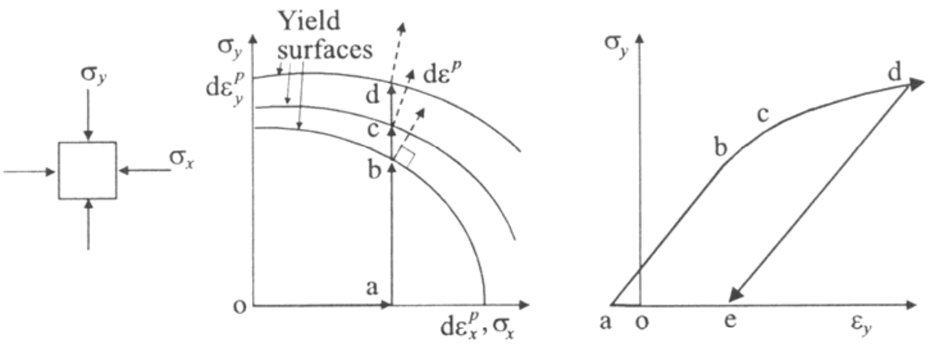


Figure 6.11: Two dimensional behaviour of a linear elastic hardening plastic material

6.11 Two dimensional behavior of a linear elastic softening plastic material

- Similar to strain hardening, except reduction of yield surface size
- Unloading – elastic; re-loading to *c* – plastic strain again

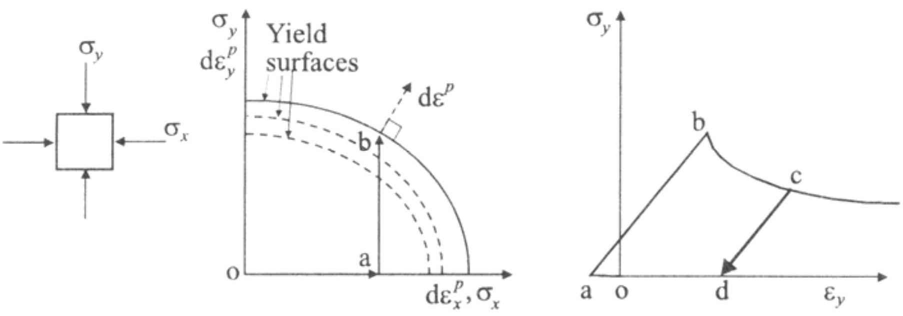


Figure 6.12: Two dimensional behaviour of a linear elastic softening plastic material

6.12 Comparison with real soil behavior

- Real soil – both strain hardening and softening
- See Chapters 7 and 8

用同一个模型来描述硬化或软化

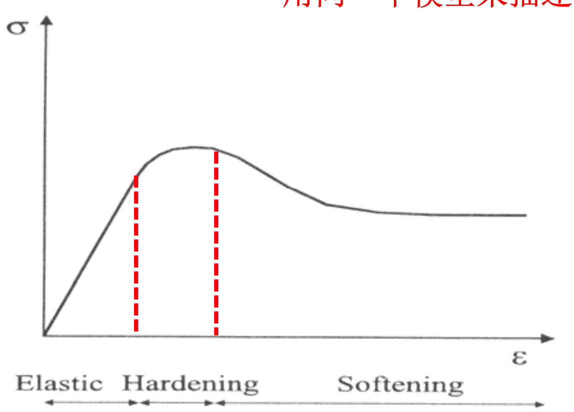


Figure 6.13: Real soil behaviour involving hardening and softening

6.13 Formulation of the elastic-plastic constitutive matrix

$$\{\Delta\sigma\} = [D^{ep}]\{\Delta\varepsilon\} \qquad (6.4)$$

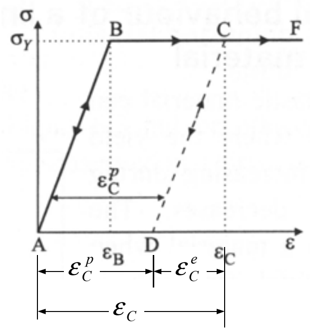
$$\{\Delta\varepsilon\} = \{\Delta\varepsilon^{e}\} + \{\Delta\varepsilon^{p}\} \qquad (6.5)$$

$$\{\Delta\sigma\} = [D]\{\Delta\varepsilon^{e}\} \longleftarrow \text{Elastic Hook's Law} \qquad (6.6)$$

$$\{\Delta\varepsilon^{e}\} = [D]^{-1}\{\Delta\sigma\} \qquad (6.7)$$

$$\{\Delta\sigma\} = [D](\{\Delta\varepsilon\} - \{\Delta\varepsilon^{p}\}) \qquad (6.8)$$

$$\{\Delta\varepsilon^{p}\} = \Lambda\left\{\frac{\partial P(\{\sigma\},\{m\})}{\partial\sigma}\right\} \qquad (6.9)$$



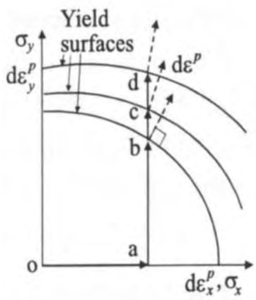
$$\{\Delta\sigma\} = [D]\{\Delta\varepsilon\} - \Lambda[D]\left\{\frac{\partial P(\{\sigma\},\{m\})}{\partial\sigma}\right\} \qquad (6.10)$$

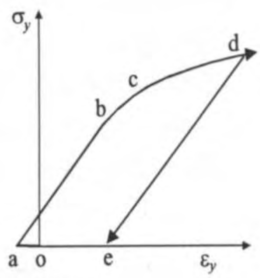
$$F\left(\{\sigma\}\,,\,\{k\}\right) = 0 \qquad (6.1)$$

$$\mathrm{d}F(\{\sigma\},\{k\}) = \left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial\sigma}\right\}^{\mathrm{T}}\{\Delta\sigma\} + \left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial k}\right\}^{\mathrm{T}}\{\Delta k\} = 0 \qquad (6.11)$$

The above equation is called:

the consistency equation (condition) – plastic strain is ALWAYS on the yield surface – dF=0





$$\mathrm{d}F(\{\sigma\},\{\boldsymbol{k}\}) = \left\{\frac{\partial F(\{\sigma\},\{\boldsymbol{k}\})}{\partial \sigma}\right\}^{\mathrm{T}}\{\Delta\sigma\} + \left\{\frac{\partial F(\{\sigma\},\{\boldsymbol{k}\})}{\partial k}\right\}^{\mathrm{T}}\{\Delta \boldsymbol{k}\} = 0 \qquad (6.11)$$

$$\{\Delta\sigma\} = -\frac{\left\{\frac{\partial F(\{\sigma\},\{\boldsymbol{k}\})}{\partial k}\right\}^{\mathrm{T}}\{\Delta \boldsymbol{k}\}}{\left\{\frac{\partial F(\{\sigma\},\{\boldsymbol{k}\})}{\partial \sigma}\right\}^{\mathrm{T}}} \qquad (6.12)$$

$$\{\Delta\sigma\} = [\boldsymbol{D}]\{\Delta\varepsilon\} - \Lambda[\boldsymbol{D}]\left\{\frac{\partial P(\{\sigma\},\{\boldsymbol{m}\})}{\partial \sigma}\right\} \qquad (6.10)$$

(6.10) = (6.12)

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$$-\frac{\left\{\frac{\partial F(\{\sigma\},\{\boldsymbol{k}\})}{\partial k}\right\}^{\mathrm{T}}\{\Delta \boldsymbol{k}\}}{\left\{\frac{\partial F(\{\sigma\},\{\boldsymbol{k}\})}{\partial \sigma}\right\}^{\mathrm{T}}} = [\boldsymbol{D}]\{\Delta\varepsilon\} - \Lambda[\boldsymbol{D}]\left\{\frac{\partial P(\{\sigma\},\{\boldsymbol{m}\})}{\partial \sigma}\right\}$$

$$-\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial k}\right\}^{T}\{\Delta k\} =$$

$$= \left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial \sigma}\right\}^{T}[D]\{\Delta\varepsilon\} - \Lambda\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial \sigma}\right\}^{T}[D]\left\{\frac{\partial P(\{\sigma\},\{m\})}{\partial \sigma}\right\}$$

$$\Lambda\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial \sigma}\right\}^{T}[D]\left\{\frac{\partial P(\{\sigma\},\{m\})}{\partial \sigma}\right\} - \frac{\Lambda}{\Lambda}\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial k}\right\}^{T}\{\Delta k\} =$$

$$= \left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial \sigma}\right\}^{T}[D]\{\Delta\varepsilon\}$$

Solve for Λ:

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Combine (6.10) and (6.12):

$$\Lambda = \frac{\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial \sigma}\right\}^{\mathrm{T}}[D]\{\Delta\varepsilon\}}{\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial \sigma}\right\}^{\mathrm{T}}[D]\left\{\frac{\partial P(\{\sigma\},\{m\})}{\partial \sigma}\right\}+A} \quad (6.13)$$

$$A = -\frac{1}{\Lambda}\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial k}\right\}^{\mathrm{T}}\{\Delta k\} \quad (6.14)$$

(6.13) into (6.10):

$$\{\Delta\sigma\} = [D]\{\Delta\varepsilon\} - \Lambda[D]\left\{\frac{\partial P(\{\sigma\},\{m\})}{\partial \sigma}\right\} \quad (6.10)$$

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(6.13) into (6.10):

$$\{\Delta\sigma\} = [D]\{\Delta\varepsilon\} - \frac{[D]\left\{\frac{\partial P(\{\sigma\},\{m\})}{\partial \sigma}\right\}\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial \sigma}\right\}^{\mathrm{T}}[D]\{\Delta\varepsilon\}}{\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial \sigma}\right\}^{\mathrm{T}}[D]\left\{\frac{\partial P(\{\sigma\},\{m\})}{\partial \sigma}\right\}+A} \quad (6.15)$$

Compare (6.15) to (6.4) to give the elastic-plastic constitutive matrix $[D^{ep}]$:

$$\{\Delta\sigma\} = [D^{ep}]\{\Delta\varepsilon\} \quad (6.4)$$

$$[D^{ep}] = [D] - \frac{[D]\left\{\frac{\partial P(\{\sigma\},\{m\})}{\partial \sigma}\right\}\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial \sigma}\right\}^{\mathrm{T}}[D]}{\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial \sigma}\right\}^{\mathrm{T}}[D]\left\{\frac{\partial P(\{\sigma\},\{m\})}{\partial \sigma}\right\}+A} \quad (6.16)$$

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Another prove: (6.10) into (6.11):

$$\{\Delta\sigma\} = [\boldsymbol{D}]\{\Delta\varepsilon\} - \Lambda[\boldsymbol{D}]\left\{\frac{\partial P(\{\sigma\},\{\boldsymbol{m}\})}{\partial\sigma}\right\} \qquad (6.10)$$

$$\mathrm{d}F(\{\sigma\},\{\boldsymbol{k}\}) = \left\{\frac{\partial F(\{\sigma\},\{\boldsymbol{k}\})}{\partial\sigma}\right\}^{\mathrm{T}}\{\Delta\sigma\} + \left\{\frac{\partial F(\{\sigma\},\{\boldsymbol{k}\})}{\partial k}\right\}^{\mathrm{T}}\{\Delta\boldsymbol{k}\} = 0 \qquad (6.11)$$

$$\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial\sigma}\right\}^{T}\left\{[D]\{\Delta\varepsilon\} - \Lambda[D]\left\{\frac{\partial P(\{\sigma\},\{m\})}{\partial\sigma}\right\}\right\} + \left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial\sigma}\right\}\{\Delta k\} = 0$$

$$\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial\sigma}\right\}^{T}[D]\{\Delta\varepsilon\} - \Lambda\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial\sigma}\right\}^{T}[D]\left\{\frac{\partial P(\{\sigma\},\{m\})}{\partial\sigma}\right\}$$
$$+\frac{\Lambda}{\Lambda}\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial\sigma}\right\}\{\Delta k\} = 0$$

$$\Lambda = \frac{\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial\sigma}\right\}^{T}[D]\{\Delta\varepsilon\}}{\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial\sigma}\right\}^{T}[D]\left\{\frac{\partial P(\{\sigma\},\{m\})}{\partial\sigma}\right\} - \frac{1}{\Lambda}\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial\sigma}\right\}\{\Delta k\}}$$

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The above into (6.10):

$$\{\Delta\sigma\} = [\boldsymbol{D}]\{\Delta\varepsilon\} - \Lambda[\boldsymbol{D}]\left\{\frac{\partial P(\{\sigma\},\{\boldsymbol{m}\})}{\partial\sigma}\right\} \qquad (6.10)$$

$$\{\Delta\sigma\} = [D]\{\Delta\varepsilon\} - \frac{[D]\left\{\frac{\partial P(\{\sigma\},\{m\})}{\partial\sigma}\right\}\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial\sigma}\right\}^{T}[D]\{\Delta\varepsilon\}}{\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial\sigma}\right\}^{T}[D]\left\{\frac{\partial P(\{\sigma\},\{m\})}{\partial\sigma}\right\} + A}$$

$$A = -\frac{1}{\Lambda}\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial\sigma}\right\}\{\Delta k\}$$

$$\{\Delta\sigma\} = [\boldsymbol{D}^{ep}]\{\Delta\varepsilon\} \qquad (6.4)$$

$$[\boldsymbol{D}^{ep}] = [\boldsymbol{D}] - \frac{[\boldsymbol{D}]\left\{\frac{\partial P(\{\sigma\},\{\boldsymbol{m}\})}{\partial\sigma}\right\}\left\{\frac{\partial F(\{\sigma\},\{\boldsymbol{k}\})}{\partial\sigma}\right\}^{\mathrm{T}}[\boldsymbol{D}]}{\left\{\frac{\partial F(\{\sigma\},\{\boldsymbol{k}\})}{\partial\sigma}\right\}^{\mathrm{T}}[\boldsymbol{D}]\left\{\frac{\partial P(\{\sigma\},\{\boldsymbol{m}\})}{\partial\sigma}\right\} + A} \qquad (6.16)$$

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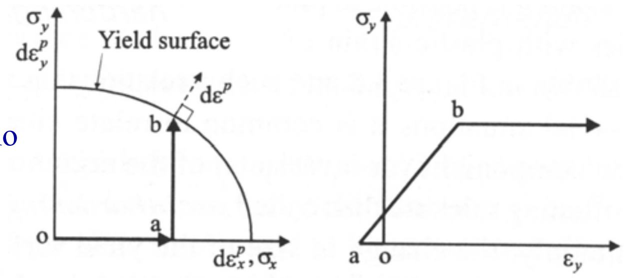
1. Perfect plasticity:

{k} – are constants, no change of the yield surface



$$\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial k}\right\}^{T} = 0 \qquad (6.17)$$

A=0 in (6.15) or (6.16)

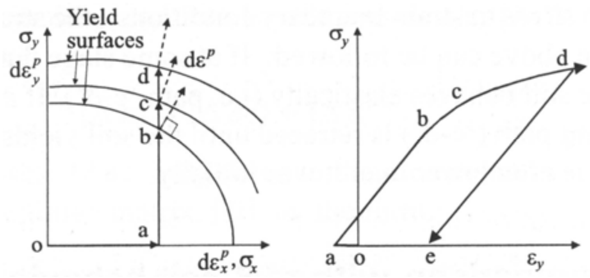
$$\{\Delta\sigma\} = [D^{ep}]\{\Delta\varepsilon\} \qquad (6.4)$$

$$[D^{ep}] = [D] - \frac{[D]\left\{\frac{\partial P(\{\sigma\},\{m\})}{\partial\sigma}\right\}\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial\sigma}\right\}^{T}[D]}{\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial\sigma}\right\}^{T}[D]\left\{\frac{\partial P(\{\sigma\},\{m\})}{\partial\sigma}\right\} + A} \qquad (6.16)$$

A=0

2. Strain hardening/ softening plasticity:

{k} – are linear function of plastic strains $\{\varepsilon^p\}$



(6.14) can be rewritten as:

$$A = -\frac{1}{\Lambda}\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial k}\right\}^{T}\frac{\partial\{k\}}{\partial\{\varepsilon^p\}}\{\Delta\varepsilon^p\} \qquad (6.18)$$

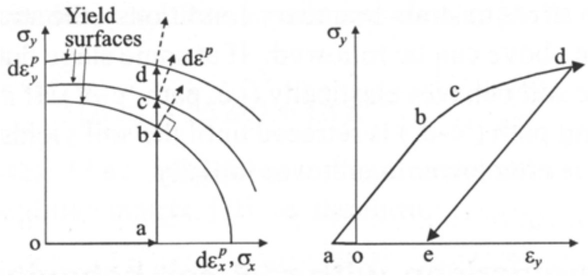
Since: $\frac{\partial\{k\}}{\partial\{\varepsilon^p\}} =$ a constant (i.e. independent of $\{\varepsilon^p\}$) (6.19)

$$\{\Delta\varepsilon^p\} = \Lambda\left\{\frac{\partial P(\{\sigma\},\{m\})}{\partial\sigma}\right\} \qquad (6.9)$$

The Λ in (6.18) is cancelled and A is determinant.

3. Working hardening/ softening plasticity:

{k} – are linear function of plastic work *W^p*



(6.14) can be rewritten as:

$$A = -\frac{1}{\Lambda}\left\{\frac{\partial F(\{\sigma\},\{k\})}{\partial \sigma}\right\}\frac{\partial\{k\}}{\partial W^p}\frac{\partial W^p}{\partial\{\varepsilon^p\}}\{\Delta\varepsilon^p\}$$

Since: $\frac{\partial\{k\}}{\partial W^p} = constant, \quad \frac{\partial W^p}{\partial\{\varepsilon^p\}} = known$

$$\{\Delta\varepsilon^p\} = \Lambda\left\{\frac{\partial P(\{\sigma\},\{\boldsymbol{m}\})}{\partial\sigma}\right\} \qquad (6.9)$$

The Λ in (6.18) is cancelled and A is determinant.

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Symmetry of $[D]^{ep}$ and associated flow rule:

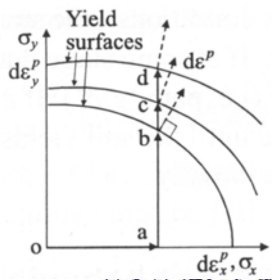
$$\{\Delta\sigma\} = [\boldsymbol{D}^{ep}]\{\Delta\varepsilon\} \qquad (6.4)$$

$$[\boldsymbol{D}^{ep}] = [\boldsymbol{D}] - \frac{[\boldsymbol{D}]\left\{\frac{\partial P(\{\sigma\},\{\boldsymbol{m}\})}{\partial\sigma}\right\}\left\{\frac{\partial F(\{\sigma\},\{\boldsymbol{k}\})}{\partial\sigma}\right\}^{\mathrm{T}}[\boldsymbol{D}]}{\left\{\frac{\partial F(\{\sigma\},\{\boldsymbol{k}\})}{\partial\sigma}\right\}^{\mathrm{T}}[\boldsymbol{D}]\left\{\frac{\partial P(\{\sigma\},\{\boldsymbol{m}\})}{\partial\sigma}\right\} + A} \qquad (6.16)$$

Elastic part *[D]* is symmetric. For *$[D]^{ep}$* to be symmetric, the condition is:

$$[\boldsymbol{D}]\left\{\frac{\partial P(\{\sigma\},\{\boldsymbol{m}\})}{\partial\sigma}\right\}\left\{\frac{\partial F(\{\sigma\},\{\boldsymbol{k}\})}{\partial\sigma}\right\}^{\mathrm{T}}[\boldsymbol{D}]$$

$$\left\{\frac{\partial P(\{\sigma\},\{\boldsymbol{m}\})}{\partial\sigma}\right\} = \left\{\frac{\partial F(\{\sigma\},\{\boldsymbol{k}\})}{\partial\sigma}\right\}$$



If associated flow rule is used: ***P=F***.

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6.14 Summary

1. Elasto-plastic theory provides probably the best framework available in which to formulate constitutive models that can realistically simulate real soil behaviour. Three types of plastic behaviour are identified: perfect plasticity and strain (or work) hardening and softening plasticity. These models assume elastic behaviour prior to yield and can therefore utilise the benefits of both elastic and plastic behaviour. 弹塑性理论框架
2. The elasto-plastic framework can incorporate both linear and nonlinear elastic behaviour. Consequently, all the models described in Chapter 5 can be incorporated.

基本假定 2个必要条件 1个补充条件

3. Elasto-plastic models are based on the assumption that the principal directions of accumulated stress and incremental plastic strain coincide. They require two essential pieces and one optional piece of information for their definition. The essential ingredients are a yield function, which separates purely elastic from elasto-plastic behaviour, and a plastic potential (or flow rule) which prescribes the direction of plastic straining. The optional ingredient is a set of hardening/softening rules which describe how the state parameters (e.g. strength) vary with plastic strain (or plastic work).
4. If the yield and plastic potential surfaces coincide, the model is said to be associated (or to satisfy the normality condition). This results in a symmetric constitutive matrix and consequently a symmetric global finite element stiffness matrix. If such a condition does not hold, both matrices are non-symmetric. This results in the use of greater computer resources, both time and memory, for finite element analyses.

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