M.Sc. in Geological Engineering: Subject No. 081803B02

Numerical Analysis in Geotechnical Engineering

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Outline of Lectures:

Lecture 1: Geotechnical analysis (Chapter 1)

Lecture 2: Finite element theory for linear materials (Chapter 2)

Tutorial 1 – Introduction to GeoStudio and SIGMA/W Examples

Lecture 3: Geotechnical considerations (Chapter 3)

Lecture 4: Real soil behaviour (Chapter 4)

Lecture 5: Elastic and hypo-elastic constitutive models (Chapter 5)

Tutorial 2 – Geotechnical Analysis using SIGMA/W and SLOPE/W

Lecture 6: Elastic-plastic behaviour (Chapter 6)

Lecture 7: Simple elastic-plastic constitutive models (Chapter 7)

Tutorial 3 – Advanced Analysis using SLOPE/W, SIGMA/W & SEEP/W

Seminar: Case Study

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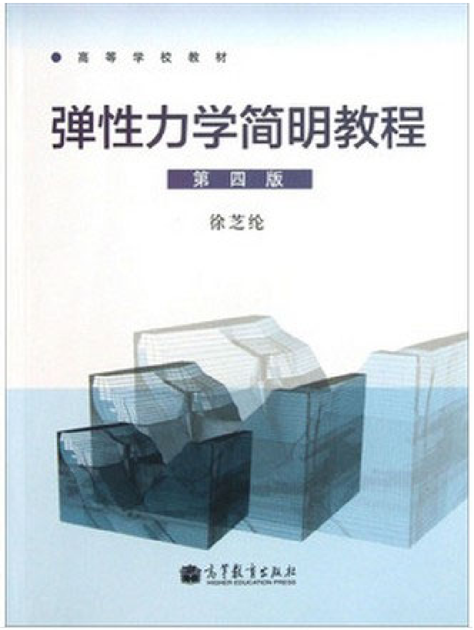
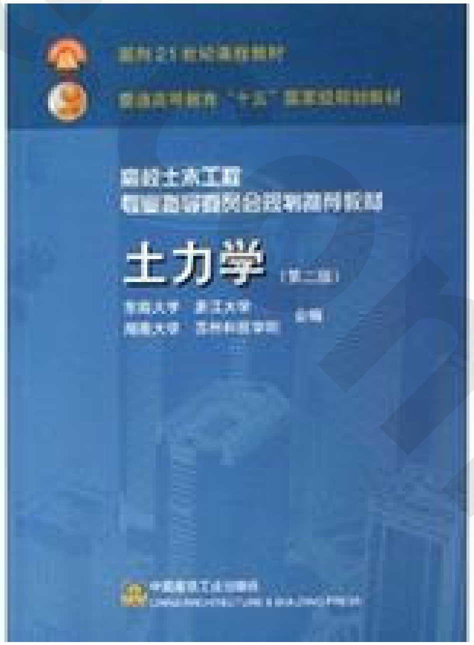
先修课程:

[1] 弹性力学

[2] 土力学

[3] 基础工程





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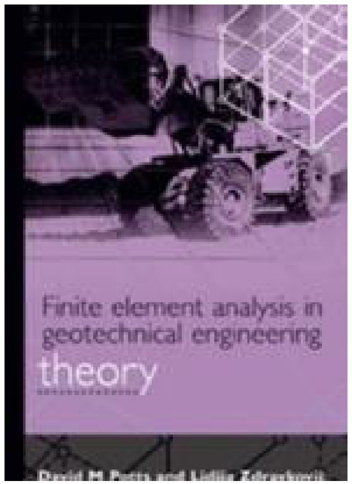
References:

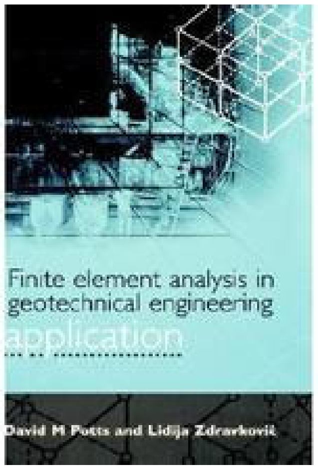
Essential References:

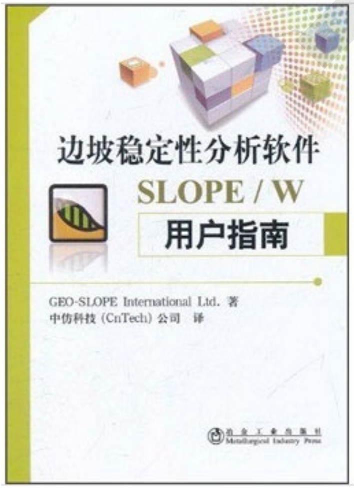
[1] David M Potts and Lidija Zdravkovic, "Finite element analysis in geotechnical engineering – theory", Thomas Telford Publishing Ltd, U.K. (1999).

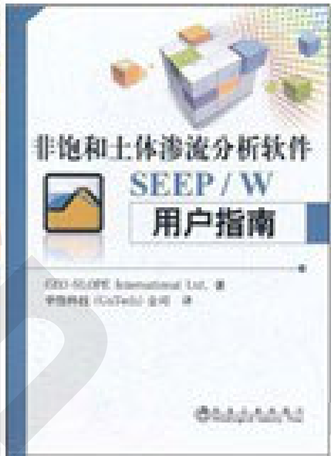
[2] David M Potts and Lidija Zdravkovic, "Finite element analysis in geotechnical engineering – application", Thomas Telford Publishing Ltd, U.K. (1999).

[3] Geo-Slope (2007), program manuals and softwares: SLOPE/W, SIGMA/W and SEEP/W.









Other References:

[1] David Muir Wood, " Soil Behaviour and Critical State Soil Mechanics", Cambridge University Press, (1990)

[2] Chen, W.F. and Mizuno, E., "Nonlinear Analysis in Soil Mechanics", Elsevier (1990)

[3] S. Helwany, "Applied Soil Mechanics with ABAQUS Applications", John Wiley & Sons, Inc. (2007)

[4] Papers published in top journals in geotechnical engineering

e.g. *Journal of Geotechnical and Geoenvironmental Engineering (ASCE), Computers and Geotechnics (Elsevier), International Journal for Numerical and Analytical Methods in Geomechanics (Wiley)*, 岩土工程学报, 岩石力学与工程学报, 岩土力学

Lecture 1: Geotechnical Analysis

1.1 Synopsis and 1.2 Introduction

1.3 Design objective

1.4 Design requirements

1.5 Theoretical considerations

1.6 Geometric idealization

1.7 Methods of analysis

1.8 Closed form solution

1.9 Simple methods

1.10 Numerical analysis

1.11 Summary

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1.1 Synopsis

不同岩土分析方法的异同

- A framework for comparison of different methods
- Advantages of numerical analysis over “conventional” approaches

1.2 Introduction

岩土结构物的特点：天然材料、相互作用（环境效应）

- Geotechnical structures – made from geological materials such as soils and rocks
- Geotechnical structures and other structures
- Geotechnical analyses – important
- What are geotechnical analyses – simple ones and sophisticated ones?

? 采用简化分析还是复杂分析方法?

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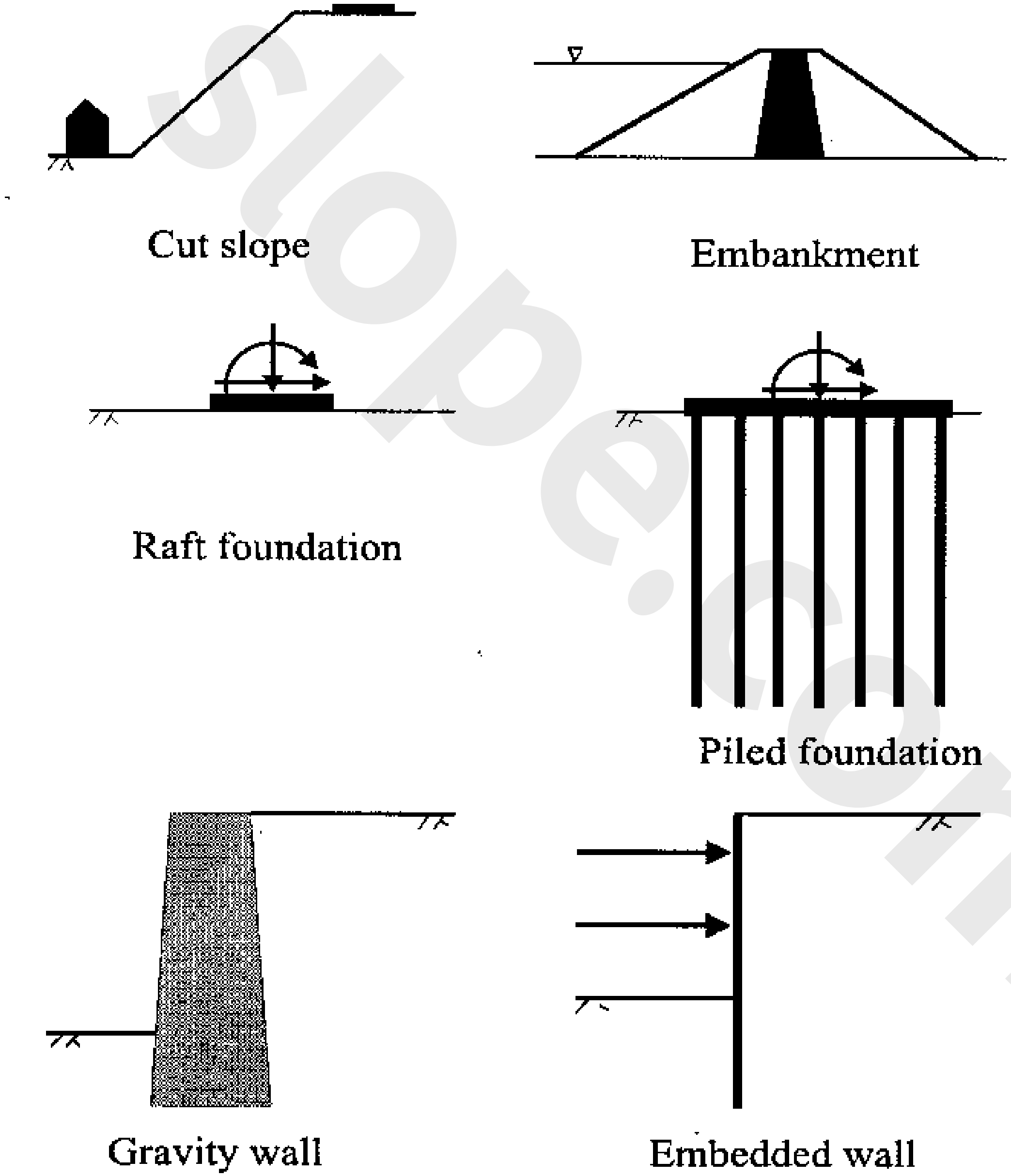

Figure 1.1: Examples of geotechnical structures

- 浅、深基础
- 基坑工程
- 边坡工程及挡土墙
- 地基处理
- ……

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1.3 Design objectives (设计目的)

- Stability (稳定性) in different forms

  - no danger of local rotational, vertical or translational failures (see Fig.1.2) and internal failures (reinforcing elements) **局部稳定性+整体稳定性**

  - no danger of overall stability failure (see Fig.1.3)

- Movements below safe limits (see Fig.1.4) **正常使用极限状态**
- Good analyses – simulating real behavior and helping engineers for better designs
- Design process more than analyses – other considerations !

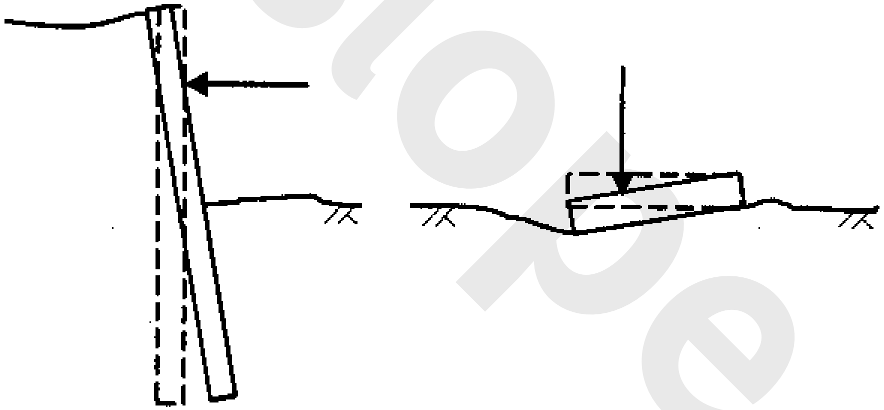


Figure 1.2: Local stability

局部稳定性

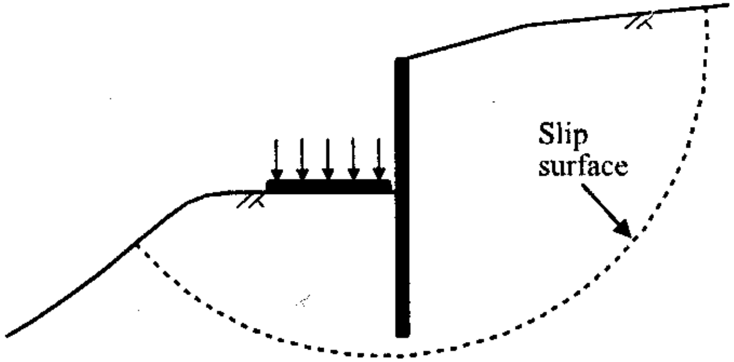
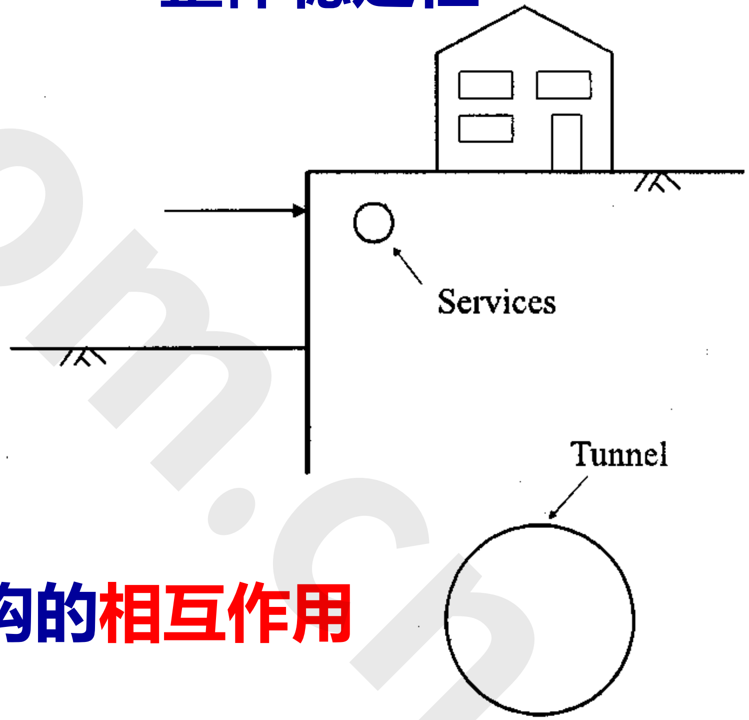


Figure 1.3: Overall stability

整体稳定性



岩土-结构的相互作用

Figure 1.4: Interaction of structures

1.4 Design requirements (设计要求)

- Before design, info is required – basic geometry, loading conditions, geo-material properties etc.
  设计相关的信息

- Geotechnical site investigation – ground conditions, properties of soil properties, water level, etc.
  岩土现场勘查

1.5 Theoretical considerations

1.5.1 Requirements for a general solution:

- **Stress equilibrium (应力平衡)**
- **Compatibility (相容性)**
- **Constitutive relationship (本构关系)**
- **Boundary (+initial) conditions (边界、初始条件)**

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1.5.2 Equilibrium（平衡）

Water flowing through a tank full of sand:

one inlet and two outlets.

The concept of stress

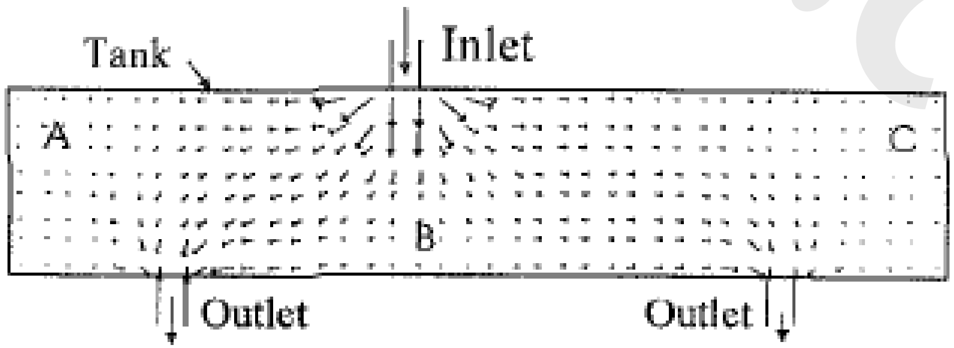


Figure 1.5: Flow trajectories

水流轨迹

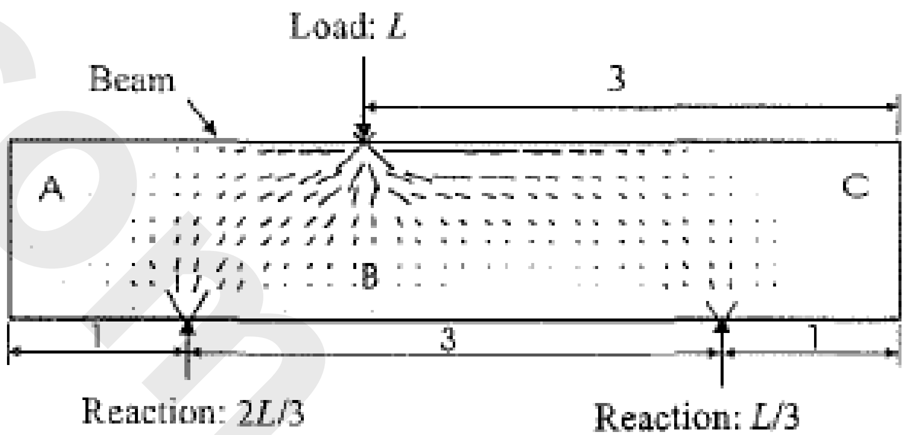


Figure 1.6: Stress trajectories

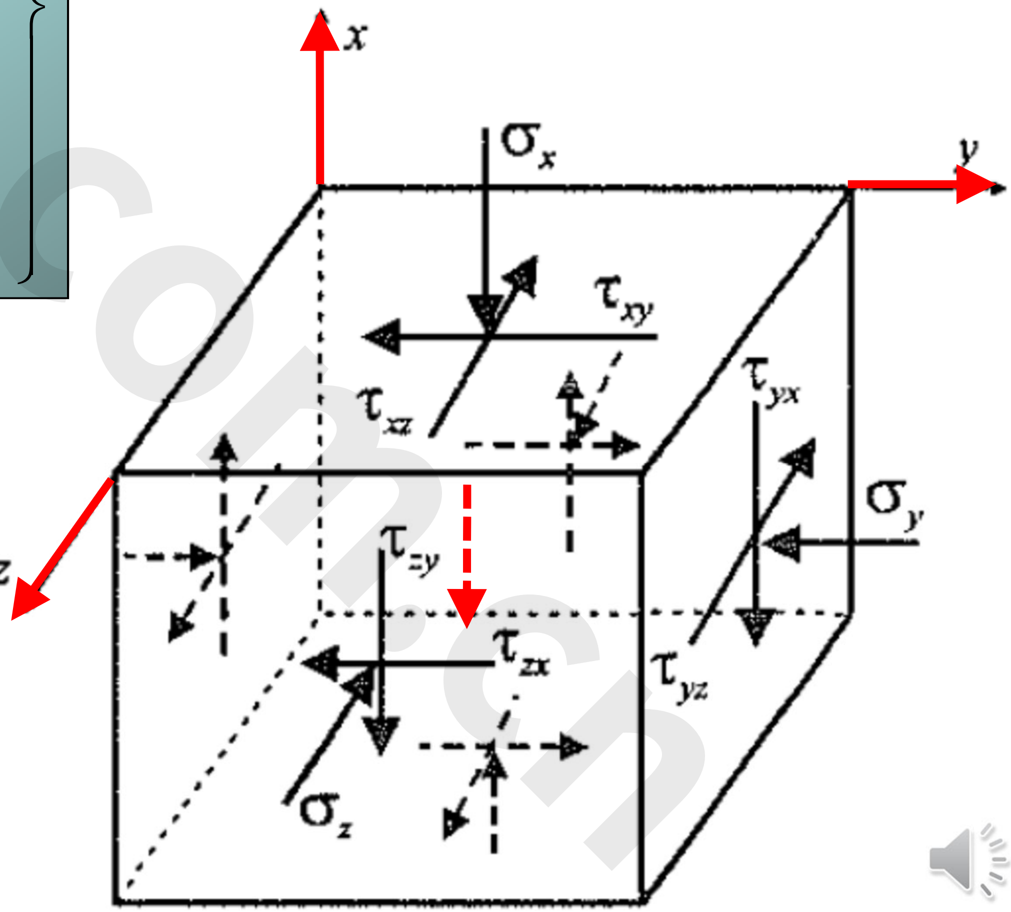
应力轨迹

(1) 3 Stress Equilibrium Equations

$$\left.\begin{aligned}\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}+\gamma=0\\ \frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{zy}}{\partial z}=0\\ \frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\tau_{yz}}{\partial y}+\frac{\partial\sigma_z}{\partial z}=0\end{aligned}\right\}$$

$$\sum F_x=0,\sum F_y=0,\sum F_z=0$$

空间直角坐标下的平衡微分方程



Note:

1.不要漏掉自重（体力）
2.正、剪应力的正负号规定
3.在边界上要满足应力和面力平衡的条件（应力边界条件）

1.5.3 Compatibility (相容性)

- 物理相容性（材料会不会重叠或裂开）
- 数学相容性

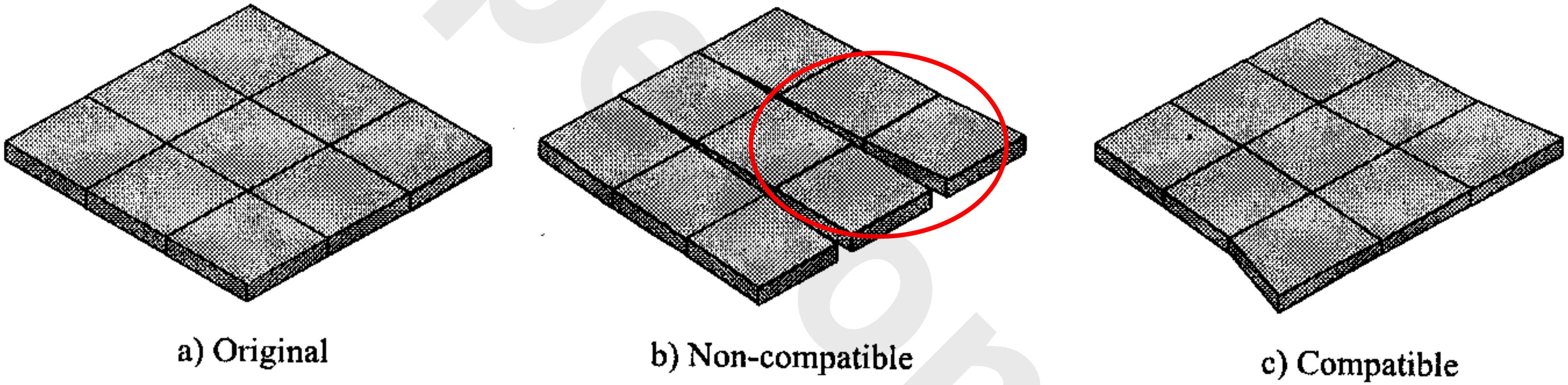


Figure 1.8: Modes of deformation

变形模式：相容还是不相容？

Overlapping or debonding?

数学相容性——几何方程

(2) 6 Strain Displace. Compatibility Equations

$$\varepsilon_x = -\frac{\partial u}{\partial x},\quad \varepsilon_y = -\frac{\partial v}{\partial y},\quad \varepsilon_z = -\frac{\partial w}{\partial z}$$

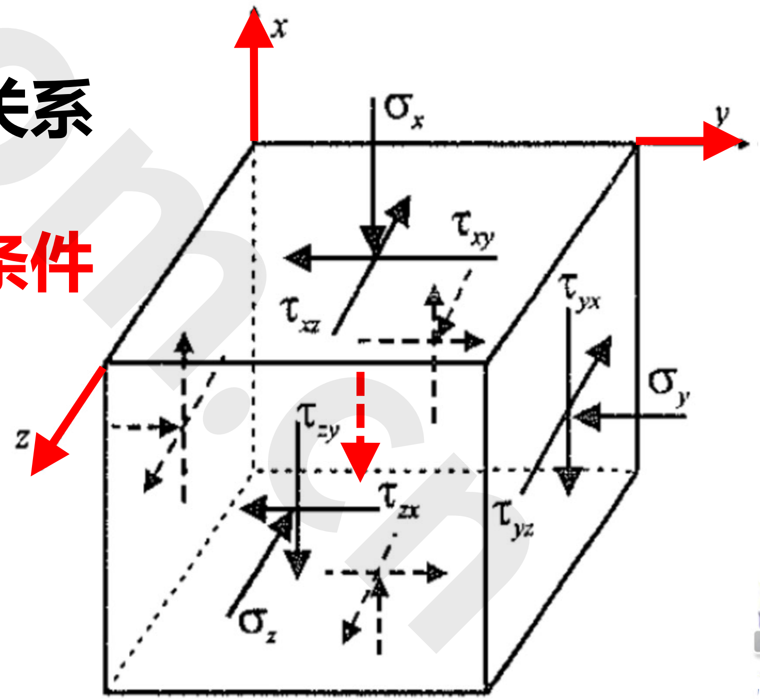
$$\gamma_{xy} = -\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y},\quad \gamma_{yz} = -\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z},\quad \gamma_{zx} = -\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}$$

Note:

1.应变和位移之间存在一阶偏导数关系

2.线应变、剪应变的正负号规定

3.在边界上要满足一定的位移边界条件



$$\varepsilon_x = -\frac{\partial u}{\partial x}, \quad \varepsilon_y = -\frac{\partial v}{\partial y}, \quad \varepsilon_z = -\frac{\partial w}{\partial z}$$

$$\gamma_{xy} = -\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \quad \gamma_{yz} = -\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \quad \gamma_{zx} = -\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}$$

6个应变分量和3个位移分量之间相互依赖

As the six strains are a function of only three displacements, they are not independent. It can be shown mathematically that for a compatible displacement field to exist, all the above components of strain and their derivatives must exist (are bounded) and be continuous to at least the second order. The displacement field must satisfy any specified displacements or restraints imposed on the boundary.

应变及其导数必须存在，且至少是二阶连续的

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1.5.4 平衡及相容条件

Now we have :

$$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}+\gamma=0$$

$$\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau_{zy}}{\partial z}=0$$

$$\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \sigma_z}{\partial z}=0$$

$$\varepsilon_x=-\frac{\partial u}{\partial x}\ ;\ \varepsilon_y=-\frac{\partial v}{\partial y}\ ;\ \varepsilon_z=-\frac{\partial w}{\partial z}$$

$$\gamma_{xy}=-\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\ ;\ \gamma_{yz}=-\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\ ;\ \gamma_{xz}=-\frac{\partial w}{\partial x}-\frac{\partial u}{\partial z}$$

Unknowns: 6 stresses + 6 strains + 3 displacements = 15
Equations: 3 equilibrium + 6 compatibility = 9

To obtain a solution therefore requires 6 more equations. These come from the constitutive relationships.

6 more equations are needed!

1.5.5 本构方程

(3) 6 Constitutive Equations (Elemental Behavior)

弹性力学中的物理方程:

$$
\left.\begin{aligned}
\varepsilon_x &= \frac{1}{E}[\sigma_x - \mu(\sigma_y + \sigma_z)] = \frac{1}{E}[(1+\mu)\sigma_x - \mu\Theta] \\
\varepsilon_y &= \frac{1}{E}[\sigma_y - \mu(\sigma_x + \sigma_z)] = \frac{1}{E}[(1+\mu)\sigma_y - \mu\Theta] \\
\varepsilon_z &= \frac{1}{E}[\sigma_z - \mu(\sigma_x + \sigma_y)] = \frac{1}{E}[(1+\mu)\sigma_z - \mu\Theta] \\
\gamma_{xy} &= \frac{\tau_{xy}}{G} \\
\gamma_{yz} &= \frac{\tau_{yz}}{G} \\
\gamma_{xz} &= \frac{\tau_{xz}}{G}
\end{aligned}\right\}
$$

$$
\left.\begin{aligned}
\sigma_x &= \frac{E}{1+\mu}\left(\frac{\mu}{1-2\mu}\theta + \varepsilon_x\right) \\
\sigma_y &= \frac{E}{1+\mu}\left(\frac{\mu}{1-2\mu}\theta + \varepsilon_y\right) \\
\sigma_z &= \frac{E}{1+\mu}\left(\frac{\mu}{1-2\mu}\theta + \varepsilon_z\right) \\
\tau_{yz} &= \frac{E}{2(1+\mu)}\gamma_{yz} = G\gamma_{yz} \\
\tau_{zx} &= \frac{E}{2(1+\mu)}\gamma_{zx} = G\gamma_{zx} \\
\tau_{xy} &= \frac{E}{2(1+\mu)}\gamma_{xy} = G\gamma_{xy}
\end{aligned}\right\}
$$

体积应力 $\Theta = \sigma_x + \sigma_y + \sigma_z$ 体积（球）应变 $\theta = \varepsilon_x + \varepsilon_y + \varepsilon_z$

本课程中的写法为增量形式(考虑非线性问题)

$$\begin{Bmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\sigma_z \\ \Delta\tau_{xy} \\ \Delta\tau_{xz} \\ \Delta\tau_{zy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ D_{31} & D_{32} & D_{33} & D_{34} & D_{35} & D_{36} \\ D_{41} & D_{42} & D_{43} & D_{44} & D_{45} & D_{46} \\ D_{51} & D_{52} & D_{53} & D_{54} & D_{55} & D_{56} \\ D_{61} & D_{62} & D_{63} & D_{64} & D_{65} & D_{66} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\varepsilon_z \\ \Delta\gamma_{xy} \\ \Delta\gamma_{xz} \\ \Delta\gamma_{zy} \end{Bmatrix}$$

or

$$\Delta\boldsymbol{\sigma} = [\boldsymbol{D}]\ \Delta\boldsymbol{\varepsilon}$$

[D]: 刚度矩阵

For a linear elastic material the [D] matrix takes the following form:

$$\begin{Bmatrix} \Delta\sigma'_x \\ \Delta\sigma'_y \\ \Delta\sigma'_z \\ \Delta\tau_{xz} \\ \Delta\tau_{yz} \\ \Delta\tau_{xy} \end{Bmatrix} = \frac{E'}{(1+\mu')(1-2\mu')} \begin{bmatrix} 1-\mu' & \mu' & \mu' & 0 & 0 & 0 \\ & 1-\mu' & \mu' & 0 & 0 & 0 \\ & & 1-\mu' & 0 & 0 & 0 \\ & & & \frac{1-2\mu'}{2} & 0 & 0 \\ & sym & & & \frac{1-2\mu'}{2} & 0 \\ & & & & & \frac{1-2\mu'}{2} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\varepsilon_z \\ \Delta\gamma_{xz} \\ \Delta\gamma_{yz} \\ \Delta\gamma_{xy} \end{Bmatrix}$$

Boundary (+initial) conditions

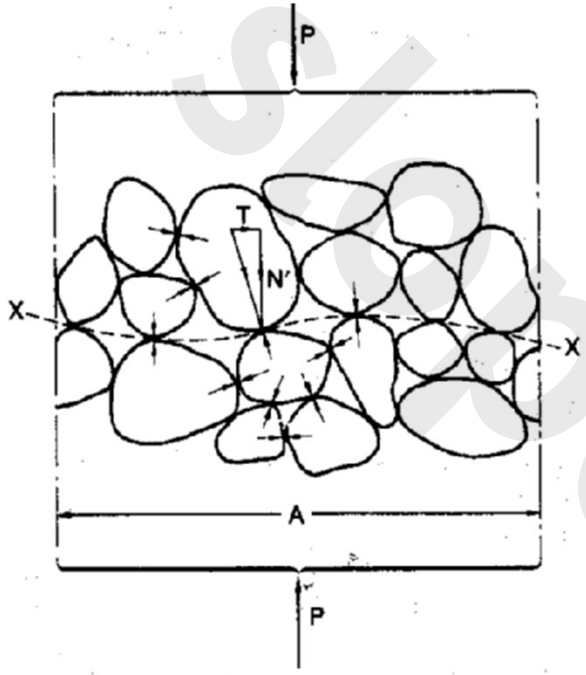
All above are needed to solve a Boundary (+initial) Value Problem

The constitutive behaviour can either be expressed in terms of total or effective stresses. If specified in terms of effective stresses, the principle of effective stress ($\sigma = \sigma' + \sigma_f$) may be invoked to obtain total stresses required for use with the equilibrium equations:

$$\Delta\boldsymbol{\sigma}' = [\boldsymbol{D}']\,\Delta\boldsymbol{\varepsilon}; \quad \Delta\boldsymbol{\sigma}_f = [\boldsymbol{D}_f]\,\Delta\boldsymbol{\varepsilon}; \quad \text{therefore } \Delta\boldsymbol{\sigma} = ([\boldsymbol{D}'] + [\boldsymbol{D}_f])\,\Delta\boldsymbol{\varepsilon} \tag{1.5}$$

where $[\boldsymbol{D}_f]$ is a constitutive relationship relating the change in pore fluid pressure to the change in strain. For undrained behaviour, the change in pore fluid pressure is related to the volumetric strain (which is small) via the bulk compressibility of the pore fluid (which is large), see Chapter 3.

? 总应力分析法，还是有效应力分析法？



Interpretation of effective stress.

$$\sigma = \frac{P}{A}$$

$$P = \Sigma N' + uA$$

$$\sigma' = \frac{\Sigma N'}{A}$$

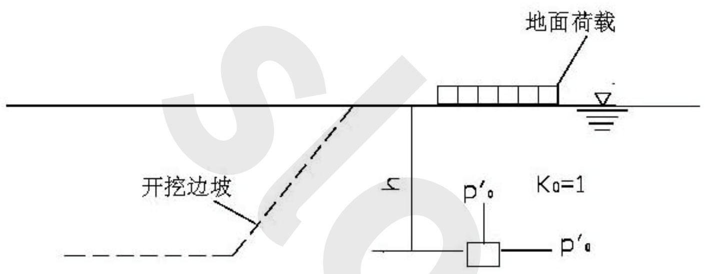
$$\frac{P}{A} = \frac{\Sigma N'}{A} + u$$

Normal stress: $\boxed{\sigma = \sigma' + u}$

Shear stress: no change

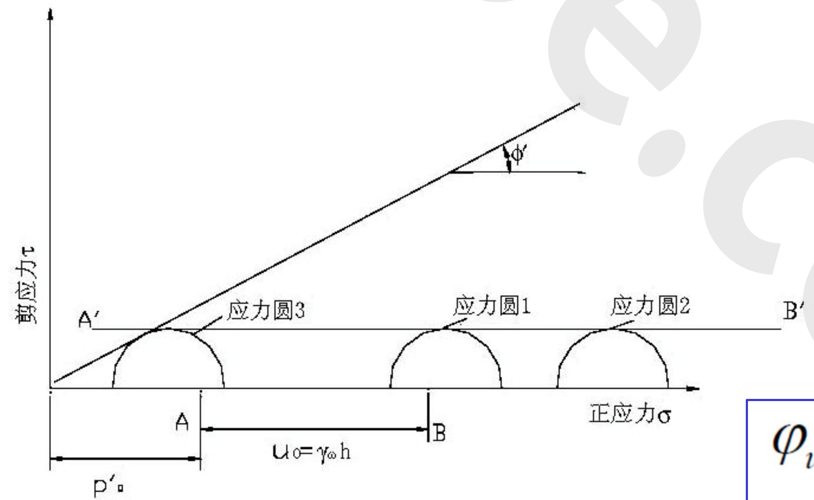
Terzaghi提出的有效应力原理

Effective stresses control: both deformation and shear resistance (or shear strength) since they reflect soil particle interaction



$$\tau_f = c_{cu} + \sigma_c' \tan\varphi_{cu}$$
$$S_u = c_{cu} + \gamma' h \tan\varphi_{cu}$$

(a)



(b)

$$\varphi_{uu} = 0$$
$$c_{uu} = S_u$$

几种特殊工况：

- **地基快速开挖；**
- **软土地基快速填筑；**
- **饱和黏性土边坡库水位骤降**

对同一种土，如果破坏时的有效法向应力和孔隙比（密实度）相同，无论什么应力路径，它们的抗剪强度相同

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关于稳定分析中考虑渗透力的问题（陈祖煜院士）

- 在进行边坡稳定分析时，首先需要解决一个**研究对象问题**。即当分析一个土体或土条的力学平衡时，是把土和水的混合体当作研究对象，还是把土骨架作为分析对象。
- 近代土力学的回答是可以把土骨架作为分析对象，也可以把包括水在内的浸水土体作为研究对象

更多阅读：边坡稳定性分析中的渗透力计算方法考证
关于“渗流作用下土坡圆弧滑动有限元计算”的讨论之二——兼论边坡稳定分析中的渗流力（李广信）

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1.6 Geometric idealisation (几何假定)

- Idealization necessary – for reasons: time saving, simplification, etc.

- Plane strain 平面应变

- Axi-symmetry 轴对称

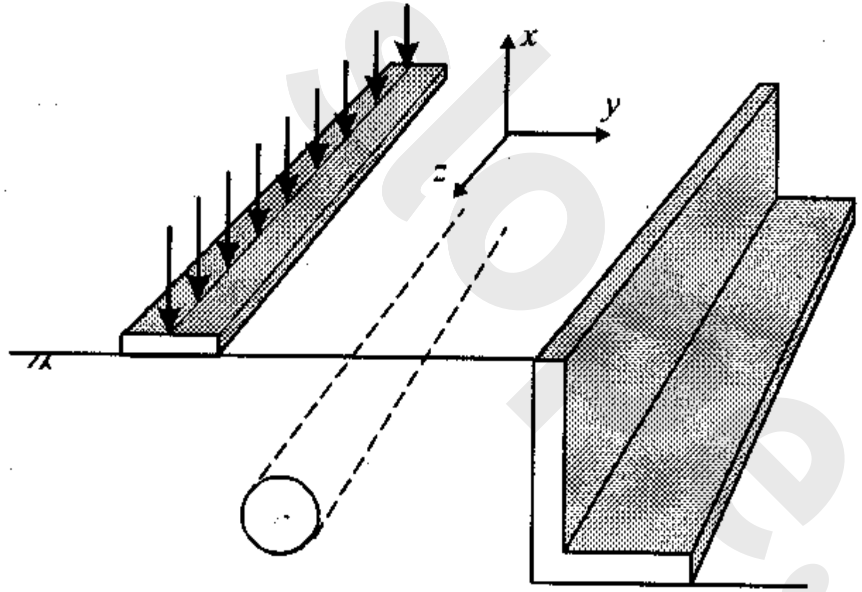
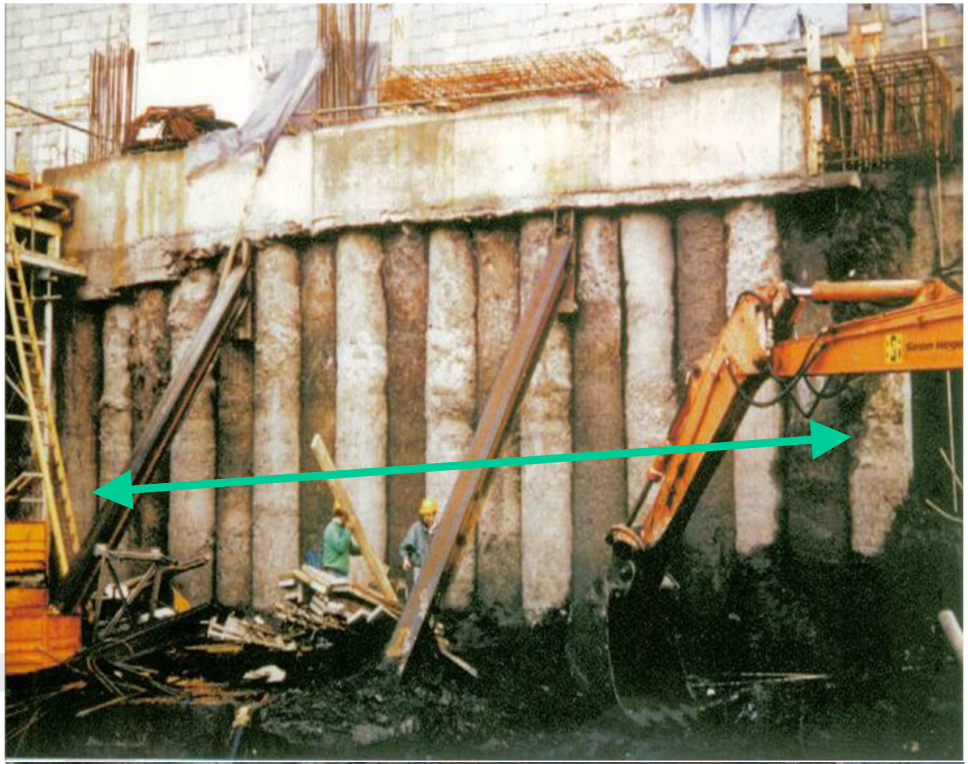
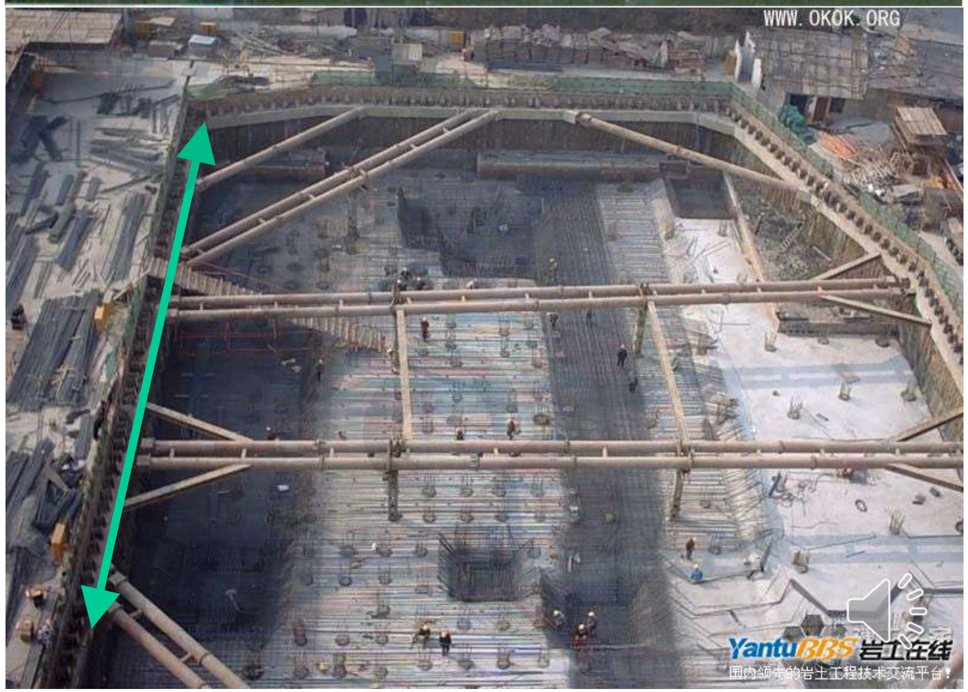


Figure 1.9: Examples of plane strain





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$$\varepsilon_z = -\frac{\partial w}{\partial z} = 0\ ;\ \gamma_{yz} = -\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 0\ ;\ \gamma_{xz} = -\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} = 0 \qquad (1.6)$$

The constitutive relationship then reduces to:

6->3/4

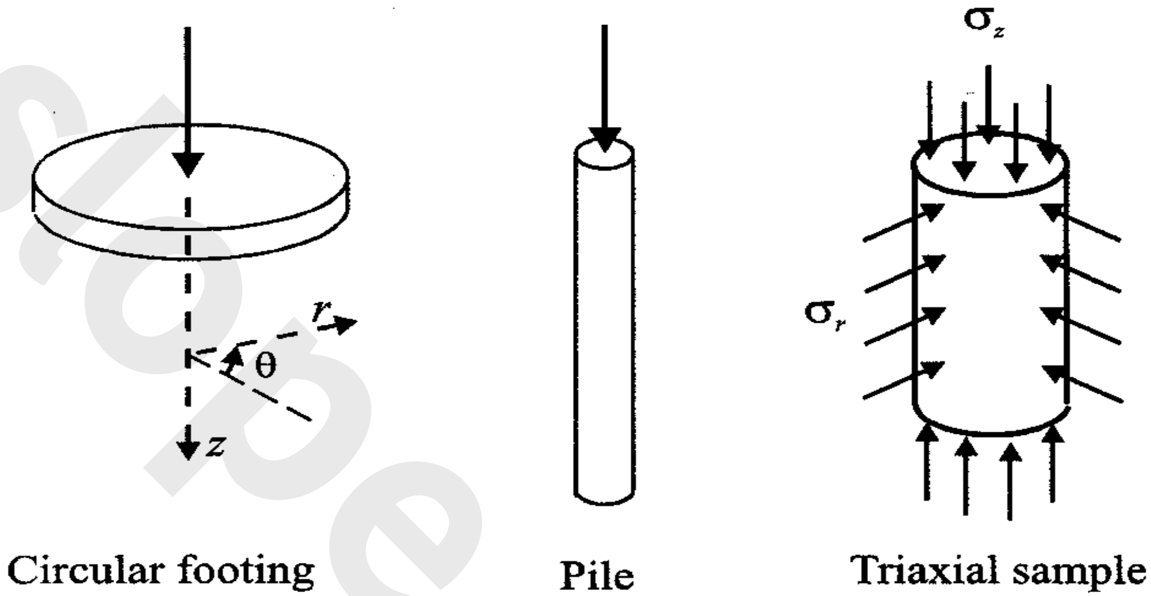
$$\begin{Bmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\sigma_z \\ \Delta\tau_{xy} \\ \Delta\tau_{xz} \\ \Delta\tau_{zy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{14} \\ D_{21} & D_{22} & D_{24} \\ D_{31} & D_{32} & D_{34} \\ D_{41} & D_{42} & D_{44} \\ D_{51} & D_{52} & D_{54} \\ D_{61} & D_{62} & D_{64} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\gamma_{xy} \end{Bmatrix} \qquad (1.7)$$

6->3

3~4个待求的应力分量

However, for elastic and the majority of material idealisations currently used to represent soil behaviour $D_{52}=D_{51}=D_{54}=D_{61}=D_{62}=D_{64}=0$, and consequently $\Delta\tau_{xz}=\Delta\tau_{zy}=0$. This results in four non-zero stress changes, $\Delta\sigma_x$, $\Delta\sigma_y$, $\Delta\sigma_z$ and $\Delta\tau_{xy}$.

It is common to consider only the stresses σ_x, σ_y and τ_{xy} when performing analysis for plane strain problems. This is acceptable if D_{11}, D_{12}, D_{14}, D_{21}, D_{22}, D_{24}, D_{41}, D_{42} and D_{44} are not dependent on σ_z. This condition is satisfied if the soil is assumed to be elastic. It is also true if the Tresca or Mohr-Coulomb failure condition is adopted (see Chapter 7) and it is assumed that the intermediate stress $\sigma_2=\sigma_z$. *Such an assumption is usually adopted for the simple analysis of geotechnical problems*. It should be noted, however, that these are special cases.



柱坐标

Figure 1.10: Examples of axi-symmetry

In this type of problem it is usual to carry out analyses using cylindrical coordinates r (radial direction), z (vertical direction) and θ (circumferential direction). Due to the symmetry, there is no displacement in the θ direction and the displacements in the r and z directions are independent of θ and therefore the strains reduce to (Timoshenko and Goodier (1951)):

$$\varepsilon_r = -\frac{\partial u}{\partial r}\ ;\ \varepsilon_z = -\frac{\partial v}{\partial z}\ ;\ \varepsilon_\theta = -\frac{u}{r}\ ;\ \gamma_{rz} = -\frac{\partial v}{\partial r} - \frac{\partial u}{\partial z}\ ;\ \gamma_{r\theta} = \gamma_{z\theta} = 0 \qquad (1.8)$$

4 non-zero stress changes: $\Delta\sigma_r, \Delta\sigma_z, \Delta\sigma_\theta, \Delta\tau_{rz}$

柱坐标下，有四个待求的非零应力分量

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1.7 Methods of analysis (分析方法)

- Meet fully or partially
  (a) equilibrium,
  (b) compatibility,
  (c) material behavior and
  (d) boundary (initial) conditions

- Methods of analysis:
  (a) Closed form solution analysis (闭合解法/解析法)
  (b) Simple analysis (简单分析法)
  (c) Numerical analysis (数值分析法)

- Comparison (see Tables 1.1 and 1.2)

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1.8 Closed form solutions (闭合解/解析解)

- A close form (exact) solution may be obtained for some simple cases meeting all conditions (a) to (d).

- The solution is exact in the theoretical sense, but still **approximate** for the real problem

- Close form solutions possible in two cases:
  - Assuming material behavior isotropic linear elastic
  - Geometric symmetries – 3-D reduced to one-dimensional problem such as expansion of spherical and infinitely long cylindrical cavitations of elasto-plastic continuum

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案例一：

- 半空间体边界上受法向集中力的布西内斯克解答

$$u_\rho = \frac{(1+\mu)P}{2\pi ER}\left[\frac{\rho z}{R^2} - \frac{(1-2\mu)\rho}{R+z}\right]$$

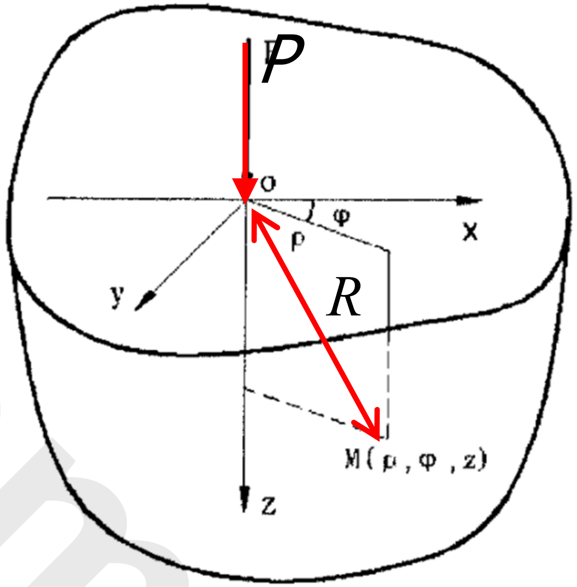
$$u_z = \frac{(1+\mu)P}{2\pi ER}\left[2(1-\mu) + \frac{z^2}{R^2}\right]$$

$$\sigma_\rho = \frac{P}{2\pi R^2}\left[\frac{(1-2\mu)R}{R+z} - \frac{3\rho^2 z}{R^3}\right]$$

$$\sigma_\varphi = \frac{(1-2\mu)P}{2\pi R^2}\left(\frac{z}{R} - \frac{R}{R+z}\right)$$

$$\sigma_z = \frac{3Pz^3}{2\pi R^5}$$

$$\tau_{z\rho} = -\frac{3P\rho z^2}{2\pi R^5}$$



其中 $R^2 = \rho^2 + z^2$

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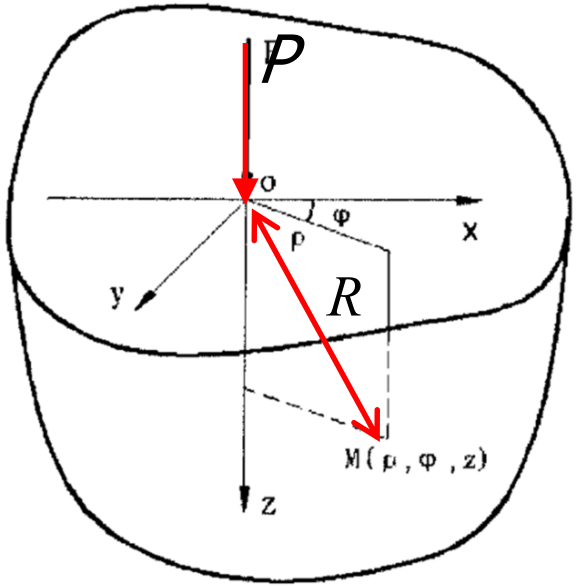
应力特征：

（1）当 $R\to\infty$, 应力 $\to 0$;

当 $R\to 0$, 应力 $\to\infty$。



（2）水平截面上的应力 σ_z 和 $\tau_{z\rho}$ 与弹性常数（弹性模量、泊松比）无关。

（3）水平截面上的全应力，指向 P 作用点 O。边界面上任一点的**沉陷**：

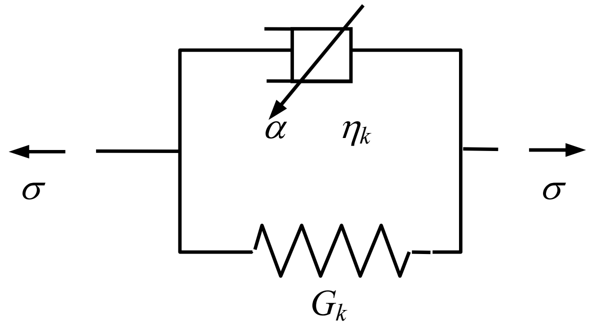
$$\eta=(u_z)_{z=0}=\frac{F(1-\mu^2)}{\pi E r\rho}$$

案例二：

- 竖向荷载作用下的分数导数型黏弹性地基沉降分析

Kelvin-Voigt model

$$\sigma(t) = E_0\varepsilon(t) + E_1 \frac{d\varepsilon(t)}{dt}$$



Three-parameter fractional Kelvin-Voigt model

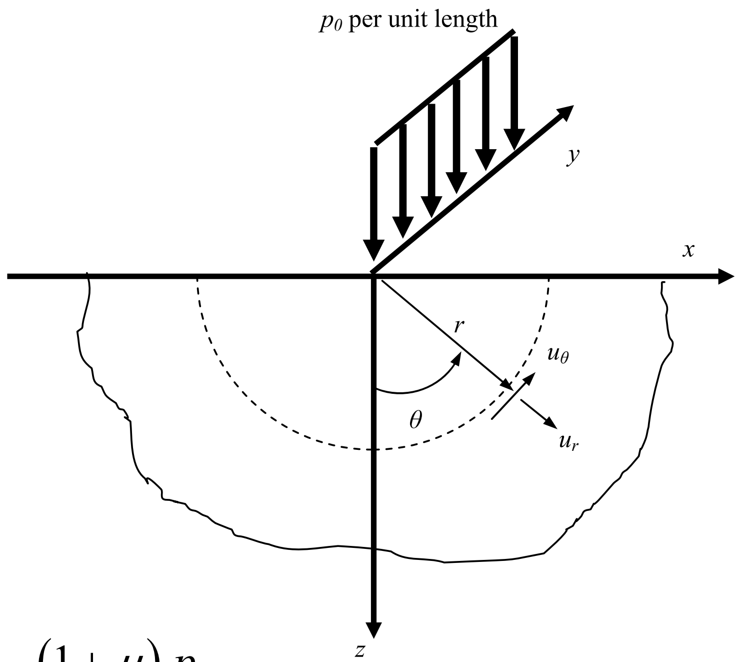
A fractional Kelvin-Voigt model

$$\sigma(t) = E_0\varepsilon(t) + E_1 \frac{d^{\alpha}\varepsilon(t)}{dt^{\alpha}}$$

$$0 < \alpha < 1$$

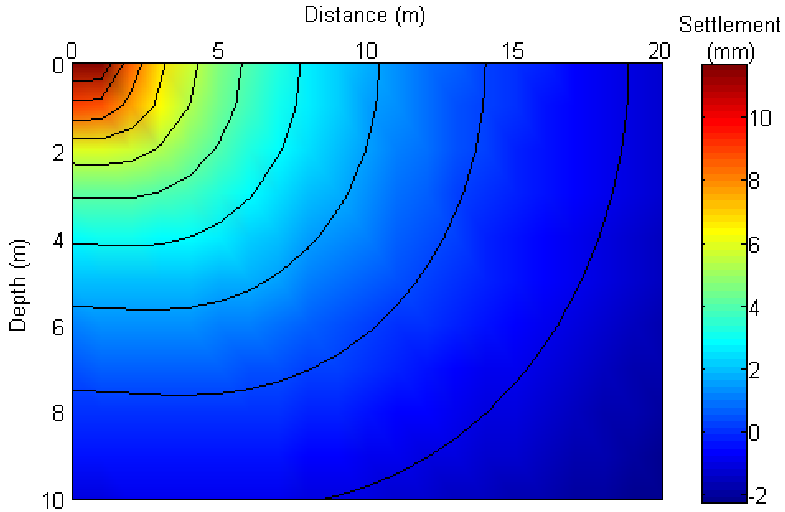
Zhu, H.-H., Liu, L.C., Pei, H.F., and Shi, B. (2012). Settlement analysis of viscoelastic foundation under vertical line load using a fractional Kelvin-Voigt model. *Geomechanics and Engineering*, 4(1), 67-78. DOI: 10.12989/gae.2012.4.1.067

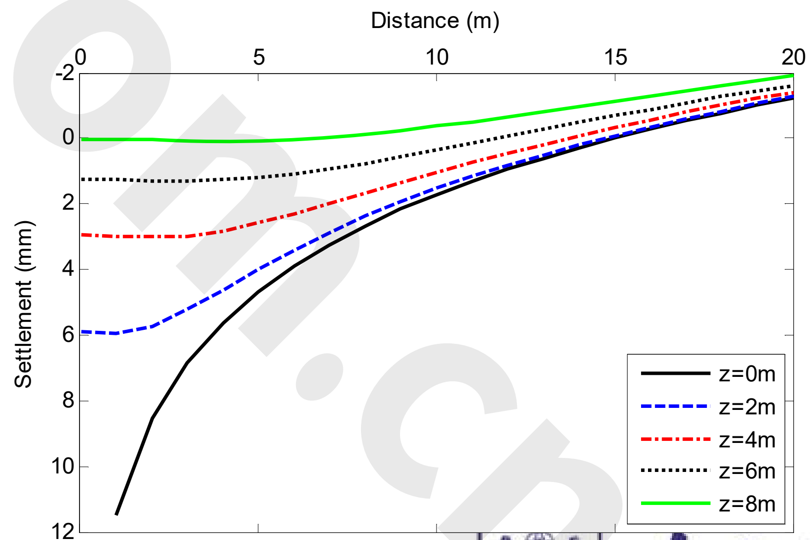
Flamant-Boussinesq elastic solution



$$u_\theta = -\frac{2p_0\left(1-\mu^2\right)}{\pi E}\sin\theta\ln\frac{r_0}{r} + \frac{(1-2\mu)(1+\mu)p_0}{\pi E}\theta\cos\theta$$

$$u_r = \frac{2p_0\left(1-\mu^2\right)}{\pi E}\cos\theta\ln\frac{r_0}{r} + \frac{(1-2\mu)(1+\mu)p_0}{\pi E}\theta\sin\theta - \frac{(1+\mu)p_0}{\pi E}\cos\theta$$





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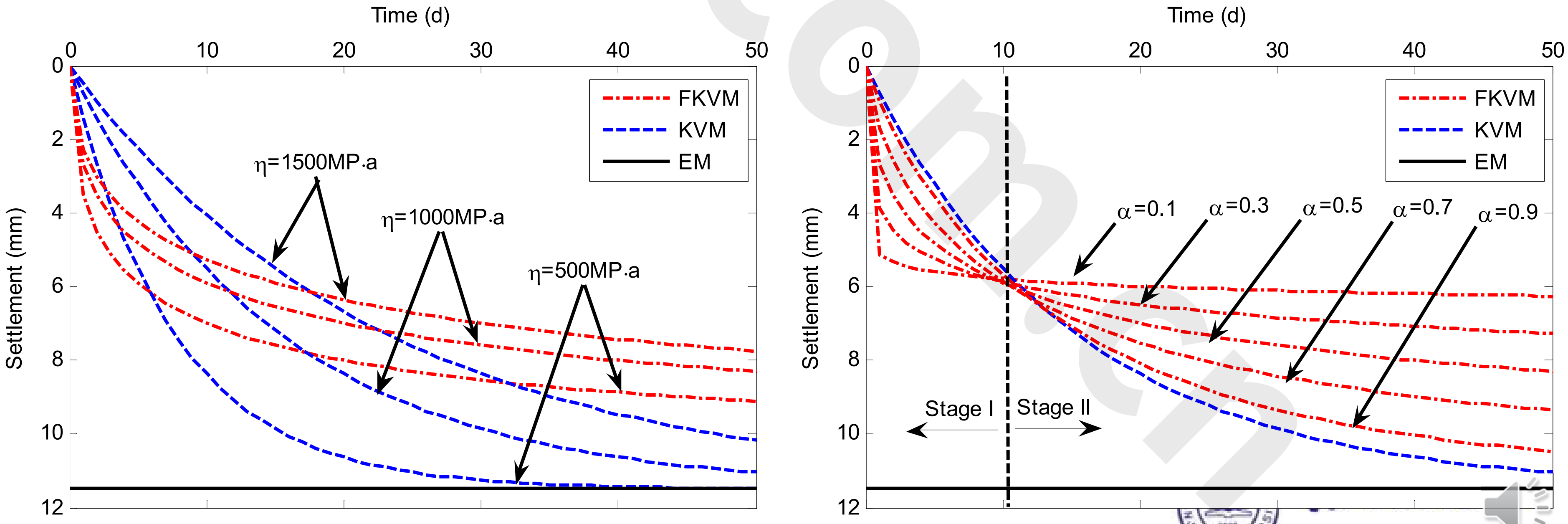
Linear visco-elastic solution

$$(u_r)_{\theta=\frac{\pi}{2}} = -\frac{3p_0}{4(3K+G)}\left(1-e^{-\frac{t}{\tau_1}}\right) \qquad (u_\theta)_{\theta=\frac{\pi}{2}} = -\frac{p_0 \ln\frac{r_0}{r}}{2\pi}\left[\frac{3}{(3K+G)}\left(1-e^{-\frac{t}{\tau_1}}\right)+\frac{1}{G}\left(1-e^{-\frac{t}{\tau_2}}\right)\right]$$

Fractional visco-elastic solution

$$(u_r)_{\theta=\frac{\pi}{2}} = -\frac{3p_0}{4(3K+G)}\left\{1-E_\alpha\left[-\left(\frac{t}{\tau_1}\right)^\alpha\right]\right\} \qquad (u_\theta)_{\theta=\frac{\pi}{2}} = -\frac{p_0 \ln\frac{r_0}{r}}{2\pi}\left\{\frac{3-3E_\alpha\left[-\left(\frac{t}{\tau_1}\right)^\alpha\right]}{(3K+G)}+\frac{1-E_\alpha\left[-\left(\frac{t}{\tau_2}\right)^\alpha\right]}{G}\right\}$$

$$E_\alpha(t)=\sum_0^\infty \frac{t^n}{\Gamma(1+\alpha n)} \qquad \tau_1 = \frac{4\eta}{3K+G} \qquad \tau_2 = \frac{\eta}{G}$$



1.9 Simple methods (简单法)

- There are 3 types of simple methods (analytical or non-analytical):
  (a) Limit equilibrium method
  (b) Stress field (characteristic line) method
  (c) Limit analysis method (upper and lower bounds)

- Simple methods are based the following simplifications (in order to get an approximate) solution meeting failure criterion:
  (a) Constrains are relaxed
  (b) Numerical approximations are introduced.

思路：解除部分约束+数值近似方法（忽略次要矛盾）

1.9.1 Limit equilibrium method (简称LEM, 极限平衡法)

- An ‘arbitrary’ failure surface (+slices) is assumed
- Equilibrium (force) is considered for the failing soil mass (+slices)
- Stress equilibrium (mass and slices) is NOT considered

- Examples:

(a) Coulomb’s wedge analysis (库仑土压力理论)

(b) Method of slices for stability analysis (计算边坡稳定性的条分法)

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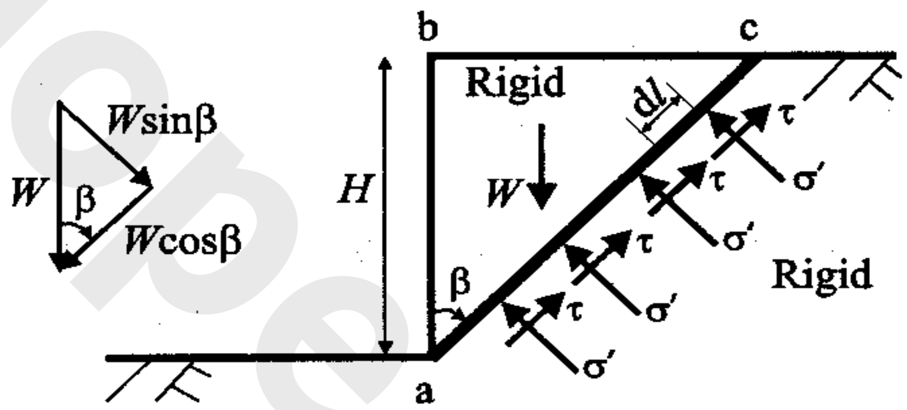
Coulomb's wedge analysis – critical height of a vertical cut:

Example: Critical height of a vertical cut



Failure criterion: $\tau = c' + \sigma' \tan \varphi'$

Figure 1.11: Failure mechanism for limit equilibrium solution

The actual distributions of σ and τ along the failure surface 'ac', presented in Figure 1.11, are unknown. However, if l is the length of the failure surface 'ac', then:

$$\int_0^l \tau \, dl = \int_0^l c' \, dl + \int_0^l \sigma' \tan\varphi' \, dl = c'l + \tan\varphi' \int_0^l \sigma' \, dl \qquad (1.9)$$

where c' and φ' are the soil's cohesion and angle of shearing resistance respectively.

Applying equilibrium to the wedge 'abc', i.e. resolving forces normal and tangential to failure surface 'ac', gives:

$$\int_0^l \sigma' \mathrm{d}l = W\sin\beta$$
$$\int_0^l \tau \,\mathrm{d}l = W\cos\beta \tag{1.10}$$

Noting that $W=\frac{1}{2}\gamma H^2\tan\beta$ and $l=H/\cos\beta$, Equations (1.9) and (1.10) can be combined to give:

$$\boxed{H = \frac{2\,c'\cos\varphi'}{\gamma\cos(\beta+\varphi')\sin\beta}} \tag{1.11}$$

The value of the angle β which produces the most conservative (lowest) value of H is obtained from $\partial H/\partial\beta=0$:

$$\frac{\partial H}{\partial \beta} = \frac{-2\,c'\cos\varphi'\cos(2\beta+\varphi')}{\gamma(\sin\beta\cos(\beta+\varphi'))^2} \tag{1.12}$$

Equation (1.12) equals zero if $\cos(2\beta+\varphi')=0$. Therefore $\boxed{\beta = \pi/4-\varphi'/2.}$

Substituting this angle into Equation (1.11) yields the *Limit equilibrium* value of H_{LE}:

$$H_{LE} = \frac{2\,c'\cos\varphi'}{\gamma\cos(\pi/4+\varphi'/2)\sin(\pi/4-\varphi'/2)} = \frac{4c'}{\gamma}\tan(\pi/4+\varphi'/2) \quad (1.13)$$

In terms of *total stress*, the equation reduces to:

$c'=S_u,\ \varphi'=0$

$$\boxed{H_{LE} = \frac{4\,S_u}{\gamma}} \quad (1.14)$$

where S_u is the undrained strength.

Note: This solution is identical to the upper bound solution obtained assuming a planar sliding surface (see Section 1.9.3). The lower bound solution gives half the above value.

在这一经典问题上：

极限平衡解答=上限解=2x下限解

后面会详细讲解

《A Short Course in Soil and Rock Engineering》：

滑面为直线型

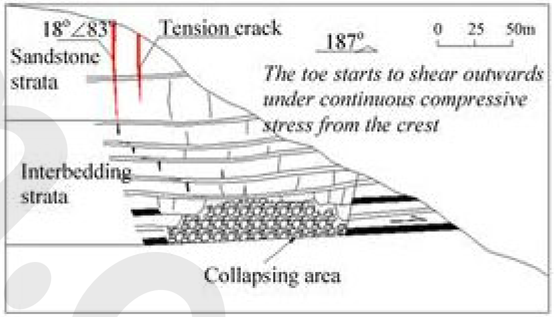
$$H_{LE} = \frac{4S_u}{\gamma}$$

滑面为圆弧型

$$H_{LE} = \frac{3.85S_u}{\gamma}$$

These predictions would be unsafe in practice because real clay soils are weak in tension.

Considering tension cracks, …



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Considering tension cracks, …

Assume the depth of tension crack $D = \frac{H_{LE}}{2}$, following Terzaghi (1943).

We get

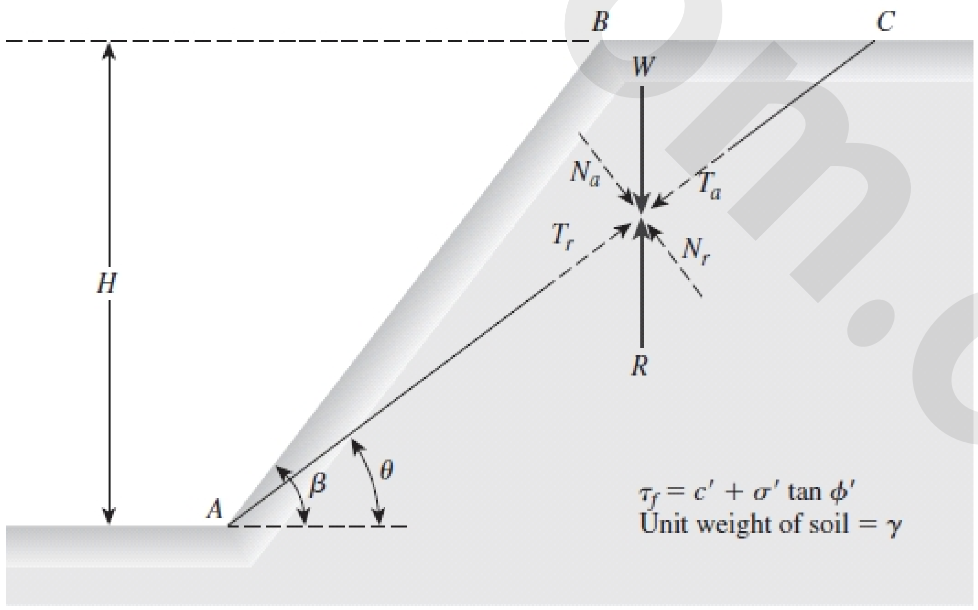
$$H_{LE} = \frac{2.67 S_u}{\gamma}$$

This gives a more realistic prediction for the height of a temporary vertical cut in clay.

坡角不为90°的边坡极限高度的极限平衡解法：**Analysis of Finite Slopes with Plane Failure Surfaces (Culmann's Method)**

from "Principle of Geotechnical Engineering" by B. M. Das

Culmann (1875)'s analysis is based on the assumption that the failure of a slope occurs along a plane when the average shearing stress tending to cause the slip is more than the shear strength of the soil. Also, the most critical plane is the one that has a minimum ratio of the average shearing stress that tends to cause failure to the shear strength of soil.



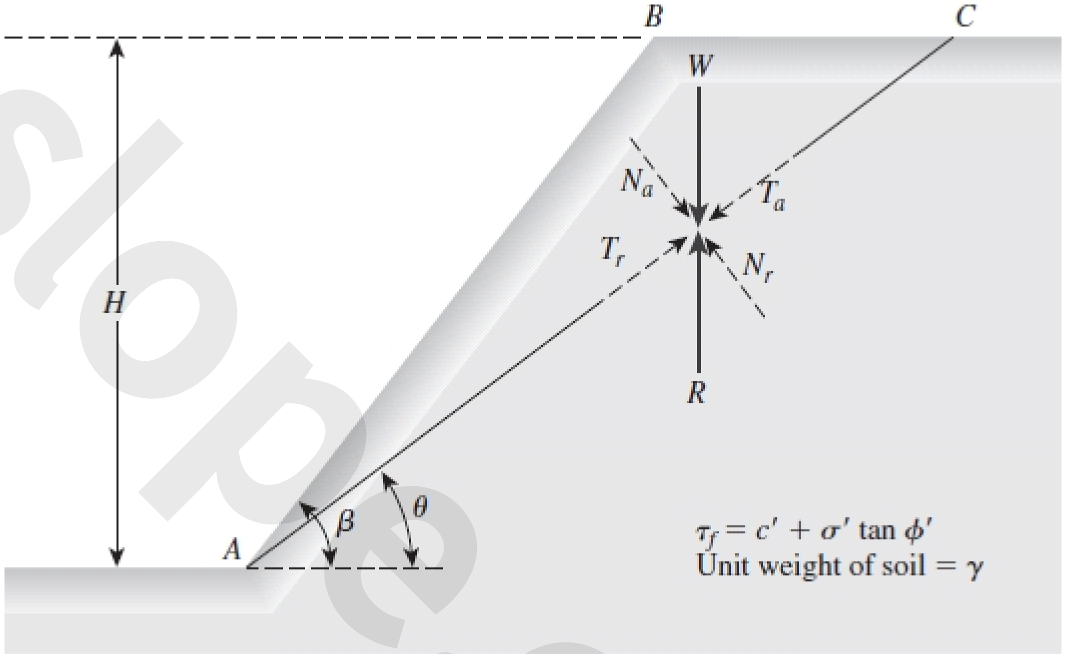


Figure 15.10 shows a slope of height *H*. The slope rises at an angle b with the horizontal. *AC* is a trial failure plane. If we consider a unit length perpendicular to the section of the slope, we find that the weight of the wedge *ABC* is equal to

$$W = \tfrac{1}{2}(H)(\overline{BC})(1)(\gamma) = \tfrac{1}{2}H(H \cot \theta - H \cot \beta)\gamma$$

$$= \frac{1}{2}\gamma H^2 \left[\frac{\sin(\beta - \theta)}{\sin \beta \sin \theta} \right] \quad (15.18)$$

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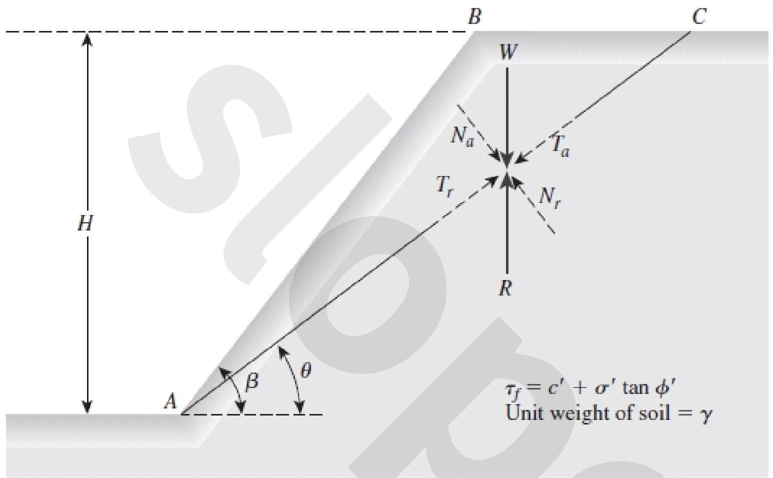


The normal and tangential components of W with respect to the plane AC are as follows.

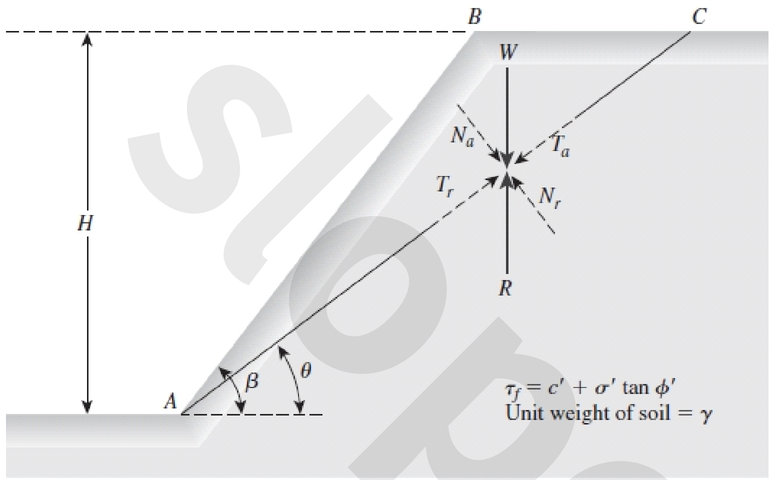
$$N_a = \text{normal component} = W \cos\theta = \frac{1}{2}\gamma H^2 \left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\cos\theta \quad (15.19)$$

$$T_a = \text{tangential component} = W \sin\theta = \frac{1}{2}\gamma H^2 \left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\sin\theta \quad (15.20)$$

The average effective normal stress and the average shear stress on the plane AC are, respectively,

$$\sigma' = \frac{N_a}{(\overline{AC})(1)} = \frac{N_a}{\left(\dfrac{H}{\sin\theta}\right)} = \frac{1}{2}\gamma H \left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\cos\theta\sin\theta \quad (15.21)$$

$$\tau = \frac{T_a}{(\overline{AC})(1)} = \frac{T_a}{\left(\dfrac{H}{\sin\theta}\right)} = \frac{1}{2}\gamma H \left[\frac{\sin(\beta-\theta)}{\sin\beta\sin\theta}\right]\sin^2\theta \quad (15.22)$$



The average resistive shearing stress developed along the plane *AC* also may be expressed as

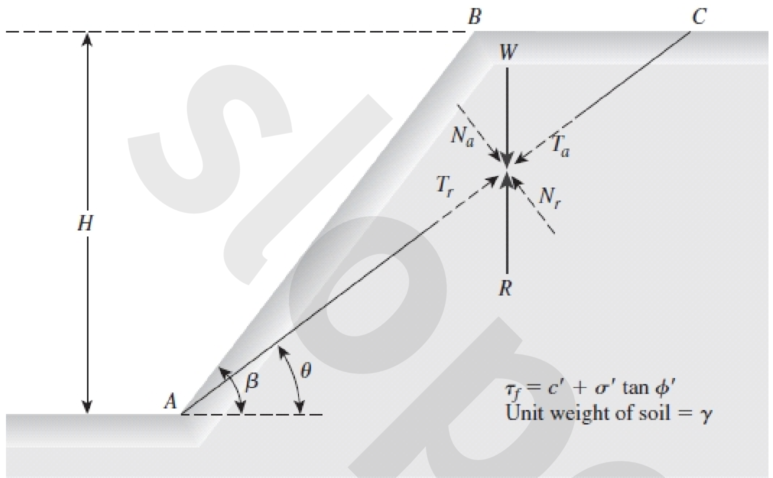
$$\tau_d = c'_d + \sigma' \tan \phi'_d$$

$$= c'_d + \frac{1}{2}\gamma H \left[\frac{\sin(\beta - \theta)}{\sin\beta \sin\theta} \right] \cos\theta \sin\theta \tan\phi'_d \qquad (15.23)$$

Now, from Eqs. (15.22) and (15.23),

$$\frac{1}{2}\gamma H \left[\frac{\sin(\beta - \theta)}{\sin\beta \sin\theta} \right] \sin^2\theta = c'_d + \frac{1}{2}\gamma H \left[\frac{\sin(\beta - \theta)}{\sin\beta \sin\theta} \right] \cos\theta \sin\theta \tan\phi'_d \qquad (15.24)$$

$$c_d = \frac{1}{2}\gamma H \left[\frac{\sin(\beta - \theta)(\sin\theta - \cos\theta \tan\phi'_d)}{\sin\beta} \right] \qquad (15.25)$$

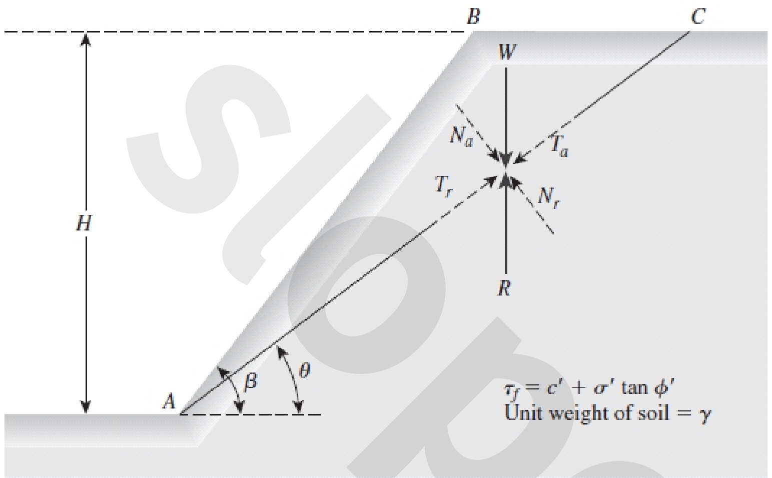


The expression in Eq. (15.25) is derived for the trial failure plane *AC*. In an effort to determine the critical failure plane, we must use the principle of maxima and minima (for a given value of ϕ'_d) to find the angle θ where the developed cohesion would be maximum. Thus, the first derivative of c_d with respect to θ is set equal to zero, or

$$\frac{\partial c'_d}{\partial \theta} = 0 \qquad (15.26)$$

Because γ, *H*, and β are constants in Eq. (15.25), we have

$$\frac{\partial}{\partial \theta}[\sin(\beta - \theta)(\sin\theta - \cos\theta \tan\phi'_d)] = 0 \qquad (15.27)$$



Solving Eq. (15.27) gives the critical value of θ, or

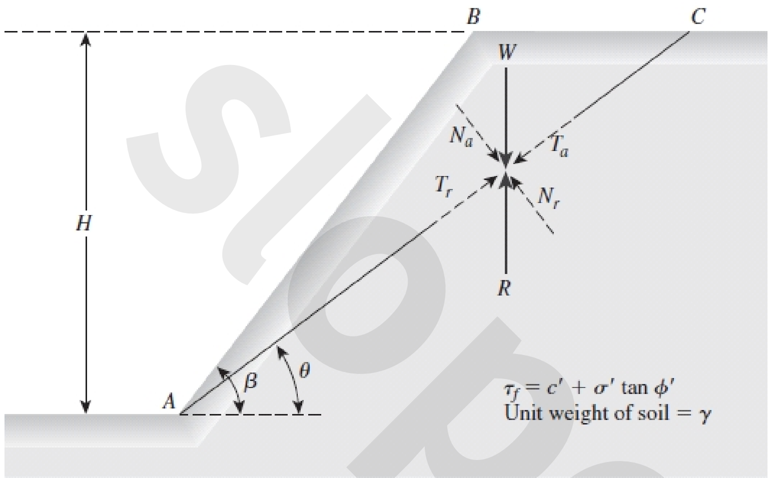
$$\theta_{cr} = \frac{\beta + \phi'_d}{2} \tag{15.28}$$

Substitution of the value of $\theta = \theta_{cr}$ into Eq. (15.25) yields

$$c'_d = \frac{\gamma H}{4}\left[\frac{1 - \cos(\beta - \phi'_d)}{\sin\beta\cos\phi'_d}\right] \tag{15.29}$$

The maximum height of the slope for which critical equilibrium occurs can be obtained by substituting $c'_d = c'$ and $\phi'_d = \phi'$ into Eq. (15.29). Thus,

$$H_{cr} = \frac{4c'}{\gamma}\left[\frac{\sin\beta\cos\phi'}{1 - \cos(\beta - \phi')}\right] \tag{15.31}$$



$$H_{cr} = \frac{4c'}{\gamma}\left[\frac{\sin\beta\cos\phi'}{1-\cos(\beta-\phi')}\right]$$

If β =90° (Vertical cut),

$$H_{cr} = \frac{4c'}{\gamma\tan\left(45^o - \frac{\phi'}{2}\right)}$$

For undrain analysis,

$$\begin{cases} c' = S_u \\ \phi' = 0 \end{cases}$$

Thus,

$$\boxed{H_{cr} = \frac{4S_u}{\gamma}}$$

The same solution!

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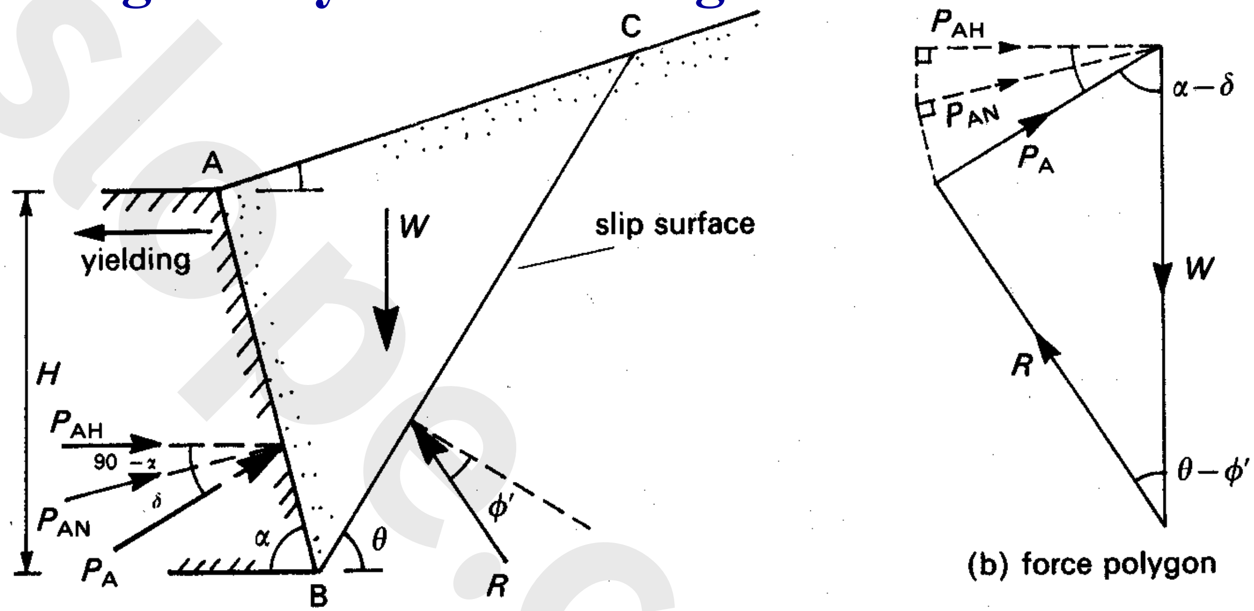
Coulomb's wedge analysis – retaining wall:



(a) forces acting on the active wedge

(b) force polygon

Fig. 8.17 Coulomb's wedge theory

maximum value of P_A is located and evaluated. First, the weight of the wedge is required and then the solution may be obtained analytically or graphically.

Weight of wedge ABC, $W=\frac{1}{2}\gamma\cdot\sin(\alpha+\beta)\cdot\overline{AB}\cdot\overline{AC}$

where $\gamma=$ unit weight of the soil

$$\overline{AC}=\text{length AC}=\overline{AB}\frac{\sin(\alpha+\theta)}{\sin(\theta-\beta)}$$

$$\overline{AB}=\text{length AB}=H/\sin\alpha$$

Worked example 8.8 *Using Coulomb's method, calculate the active thrust acting on a vertical wall of height 6.0 m due to a mass of homogeneous soil having an unsurcharged horizontal surface and the following properties.*

$c' = 0 \qquad \phi' = 30° \qquad \delta = 15° \qquad \gamma = 19\ kN/m^2$

Trial angles of the slip surface (θ) will be chosen of 56°, 58°, 60°, 62°, 64°, 66°. An analytical solution is relatively straightforward here; the model polygon of forces is shown in Fig. 8.18.

Resolving vertically: $W - R\cos(\theta - \phi') - P_A \cos(\alpha - \delta) = 0$

Resolving horizontally: $R\sin(\theta - \phi') - P_A \sin(\alpha - \delta) = 0$

From which: $$P_A = \frac{W}{\cos(\alpha - \delta) + \dfrac{\sin(\alpha - \delta)}{\tan(\theta - \phi')}}$$

Now, $\alpha = 90°$ and $\sin(\alpha - \beta) = \sin 90° = 1$; $AB = H/\sin\alpha = 6.0$ m; $\cos(\alpha - \delta) = \cos(90° - 15°) = 0.2588$ and $\sin(\alpha - \delta) = 0.9659$.

$$AC = AB\sin(\alpha + \theta)/\sin(\theta - \beta) = 6.0 \times \sin(90° + \theta)/1.0$$

Weight, $W = \frac{1}{2}\gamma \cdot \sin(\alpha + \beta) \cdot \overline{AB} \cdot \overline{AC} = 9.5 \times 1.0 \times 6.0 \times AC$

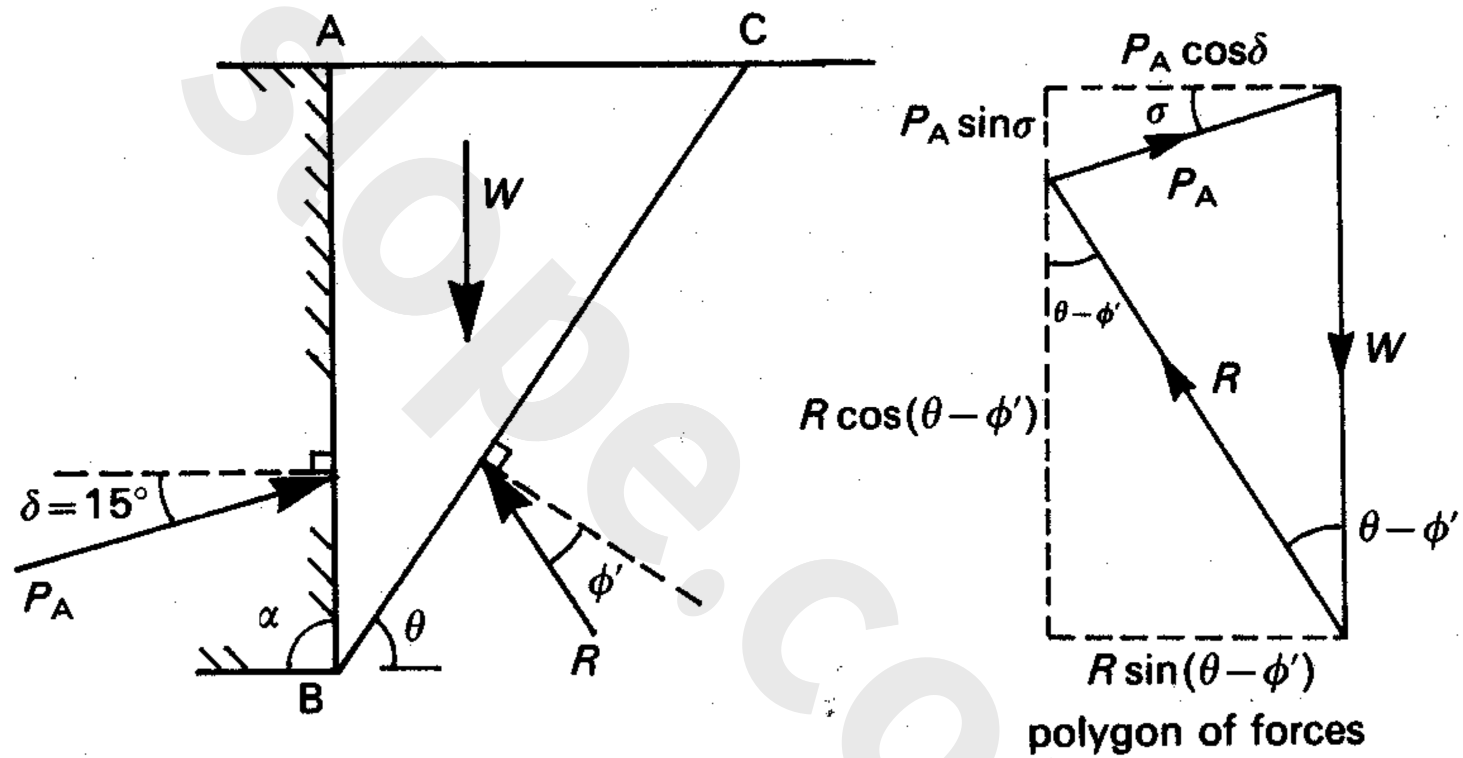


Fig. 8.18 Worked example 8.8. Polygon of forces

and

$$P_A = W/[\sin(\alpha-\delta)+\cos(\alpha-\delta)/\tan(\theta-\phi')] = W/[0.9659+0.2588/\tan(\theta-\phi')]$$

Tabulating the results:

Trial angle θ (deg)	sin(α+θ)	AC (m)	Weight W (kN/m)	tan(θ−φ′)	P_A (kN/m)
56	0.5592	3.355	191.2	0.4877	85.4
58	0.5299	3.180	181.2	0.5317	87.3
60	0.5000	3.000	171.0	0.5774	88.5
62	0.4695	2.817	160.6	0.6249	**89.0**
64	0.4384	2.630	149.9	0.6745	88.7
66	0.4067	2.440	139.1	0.7265	87.6
68	0.3746	2.248	128.1	0.7813	85.7

The variation in the magnitude of P_A with the trial angle is shown plotted in Fig. 8.19; the maximum value is the critical value: $P_A = 89.0$ kN/m and critical angle $\theta = 62°$.

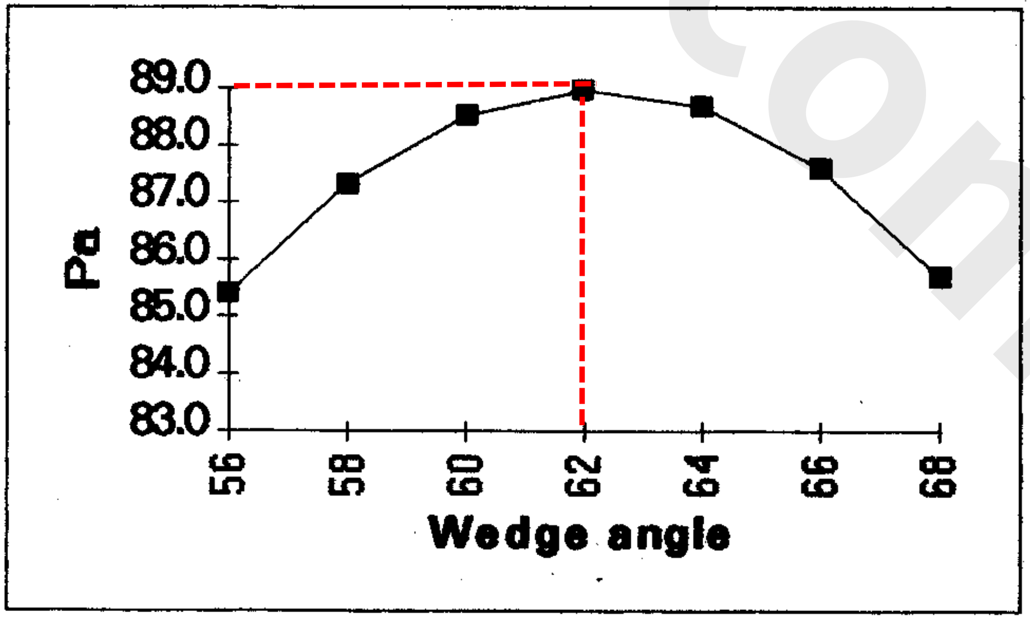


Fig. 8.19

(a) Drained conditions ($c'=0$)

Referring again to Fig. 8.17, a general solution can be obtained for the maximum active thrust in a form using an earth pressure coefficient (K_a). From the geometry of the force polygon:

$$P_A = \frac{W\sin(\theta-\phi')}{\sin[(\alpha-\delta)+(\theta-\phi')]}$$

and

$$W = \tfrac{1}{2}\gamma\sin(\alpha+\beta)\cdot\overline{AB}\cdot\overline{AC} = \tfrac{1}{2}\gamma H^2 \times F\{\alpha, \beta, \theta\}$$

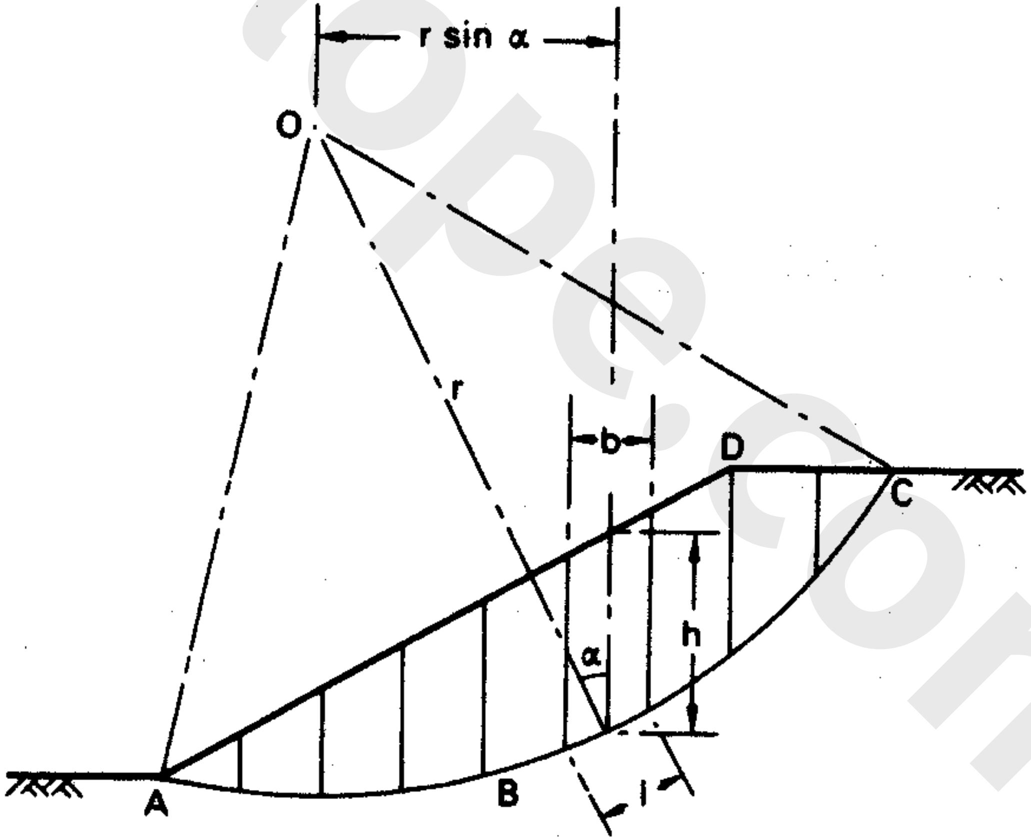
After substituting for W, differentiating and putting $\partial P/\partial\theta = 0$, the maximum value can be written:

$$\boxed{P_A = \tfrac{1}{2}\gamma H^2 \cdot \frac{K_a}{\sin\alpha\cdot\cos\delta}} \qquad [8.21]$$

where $$K_a = \frac{\sin^2(\alpha+\phi')\cdot\cos\delta}{\sin\alpha\cdot\sin(\alpha-\delta)\left[1+\sqrt{\dfrac{\sin(\phi'+\delta)\cdot\sin(\phi'-\beta)}{\sin(\alpha-\delta)\cdot\sin(\alpha+\beta)}}\right]^2} \qquad [8.22]$$

Thrust component **normal** to the wall, $P_{AN} = \tfrac{1}{2}\gamma H^2 \dfrac{K_a}{\sin\alpha} = P_A\cdot\cos\delta$ [8.23]

Method of slices for slope stability analysis – 边坡稳定性分析的条分法



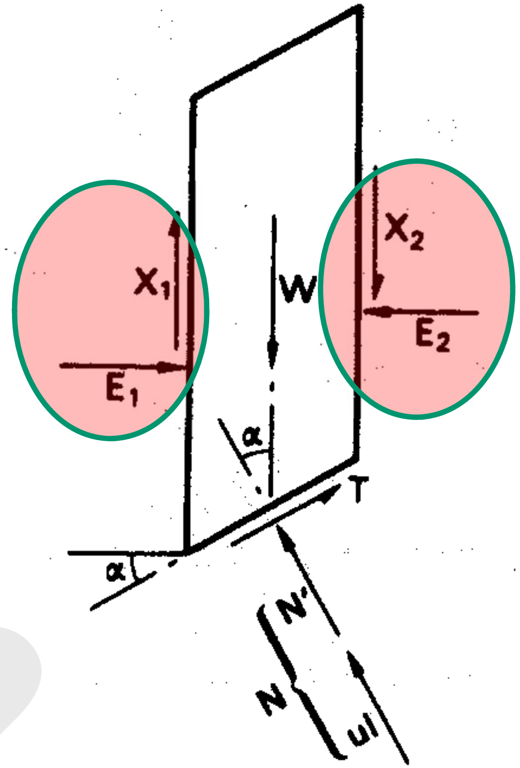


Figure 9.5 The method of slices.

$$F = \frac{\tau_f}{\tau_m}$$

$$\Sigma Tr = \Sigma Wr \sin\alpha$$

Swedish slice method（瑞典条分法）

Fellenius (1927)

Now

$$T = \tau_m l = \frac{\tau_f}{F} l$$

$$\therefore \sum \frac{\tau_f}{F} l = \Sigma W \sin\alpha$$

$$\therefore F = \frac{\Sigma \tau_f l}{\Sigma W \sin\alpha}$$

忽略土条侧面的作用力X_i、E_i

For an effective stress analysis (in terms of tangent parameters c' and ϕ'):

$$\boxed{F = \frac{\Sigma(c' + \sigma' \tan\phi')l}{\Sigma W \sin\alpha}}$$

or

$$F = \frac{c' L_a + \tan\phi' \Sigma N'}{\Sigma W \sin\alpha} \tag{9.3a}$$

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The Bishop routine solution

In this solution it is assumed that the resultant forces on the sides of the slices are horizontal, i.e.

$$\boxed{X_1 - X_2 = 0}$$

Bishop's simplified method （简化毕肖普法）

For equilibrium the shear force on the base of any slice is

$$T = \frac{1}{F}(c'l + N' \tan\phi')$$

Resolving forces in the vertical direction:

$$W = N' \cos\alpha + ul\cos\alpha + \frac{c'l}{F}\sin\alpha + \frac{N'}{F}\tan\phi' \sin\alpha$$

$$\therefore\ N' = \frac{[W - (c'l/F)\sin\alpha - ul\cos\alpha]}{[\cos\alpha + (\tan\phi' \sin\alpha)/F]} \qquad (9.6)$$

It is convenient to substitute

$$l = b\sec\alpha$$

From Equation 9.3(a), after some rearrangement,

$$\boxed{F = \frac{1}{\Sigma W \sin\alpha}\sum\left[\{c'b + (W - ub)\tan\phi'\}\frac{\sec\alpha}{1 + (\tan\alpha\tan\phi'/F)}\right]} \qquad (9.7)$$

A brief introduction to the history of slope stability analysis methods (Bromhead, 1985)

❖ Bishop's paper was presented at a conference a year before it appeared in print (1955, Geotechnique).

❖Separate researches led Janbu and Kenney to the same result (Kenney working under Bishop at Imperial College)

❖ Janbu published first, in 1955, but in an incorrect form, and Kenney's thesis appeared the following year.

❖ Subsequently, Janbu republished a corrected form of the equations, but in a form relatively inaccessible to English readers.

❖ In the meantime, Bishop had persuaded Price, who had been one of the authors of the first stability analysis computer program (see Little & Price, 1958), to try to program Kenney's equations for non-circular slips.

❖ It was then found that the basic equations cold give rise to numerical problems when evaluated to high precision that would not appear at slide-rule accuracy

❖ **Price and Morgenstern** (Morgenstern and Price, 1965, 1967) then developed a more sophisticated method (again at **Imperial College**)

❖This time, they were secure in the knowledge that complexity in computation was no longer a bar to widespread use of a method because of the growing availability of computers.

❖ **Janbu** developed his method further, and published his generalized procedure of slices in 1973, and a number of other methods also appeared in print throughout the late 1960s and 1970s. 通用条分法

❖陈祖煜院士80年代在加拿大留学期间跟Morgenstern教授研究边坡稳定性分析方法，蔡美峰院士也做过很多贡献

1.9.2 Stress field (characteristic line)) method (应力场滑移线法)

- This approach assumes a soil is at point of failure
- A solution is obtained meeting the Mohr-Coulomb failure, stress equilibrium, boundary conditions
- The solution is NOT a lower bound solution because only a partial stress field is considered (or in equilibrium), not the whole mass.
- Examples :

----Earth pressures obtained by Sokolovski (1960, 1965)

----Rankine active and passive stress fields (土力学课程已讲解)

----地基承载力的解析解

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gives the following:

Equilibrium equations:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$
$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = \gamma \qquad (1.15)$$

Mohr-Coulomb failure criterion (from Figure 1.12):

$$\sigma_1' - \sigma_3' = 2c'\cos\varphi' + (\sigma_1' + \sigma_3')\sin\varphi' \qquad (1.16)$$

Noting that:

$$s = c'\cot\varphi' + \tfrac{1}{2}(\sigma_1' + \sigma_3')$$
$$= c'\cot\varphi' + \tfrac{1}{2}(\sigma_x' + \sigma_y')$$

$$t = \tfrac{1}{2}(\sigma_1' - \sigma_3') = [\tfrac{1}{4}(\sigma_x' - \sigma_y')^2 + \tau_{xy}^2]^{0.5}$$

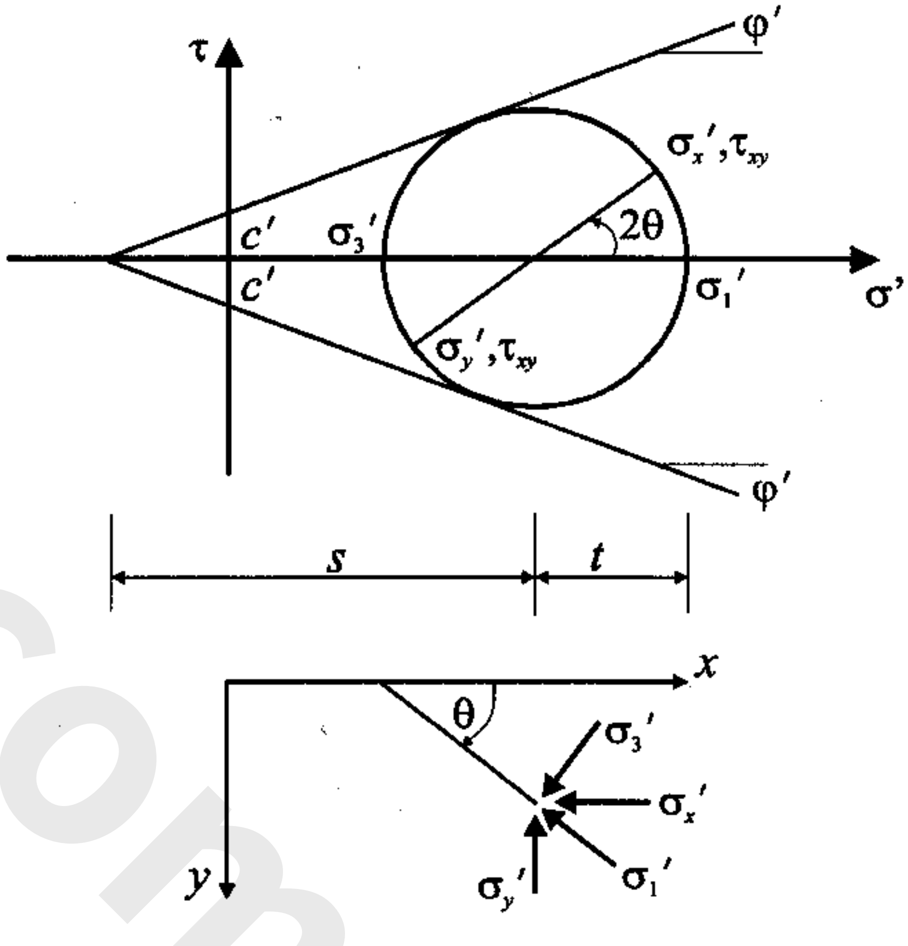

Figure 1.12: Mohr's circle of stress

and substituting in Equation (1.16), gives the following alternative equations for the Mohr-Coulomb criterion:

$$t = s \sin\varphi' \tag{1.17}$$

$$[\tfrac{1}{4}(\sigma_x' - \sigma_y')^2 + \tau_{xy}^2]^{0.5} = [c' \cot\varphi' + \tfrac{1}{2}(\sigma_x' + \sigma_y')] \sin\varphi' \tag{1.18}$$

The equilibrium Equations (1.15) and the failure criterion (1.18) provide three equations in terms of three unknowns. It is therefore theoretically possible to obtain a solution. Combining the above equations gives:

$$\begin{aligned}
&(1+\sin\varphi' \cos 2\theta)\frac{\partial s}{\partial x} + \sin\varphi' \sin 2\theta \frac{\partial s}{\partial y} + 2s \sin\varphi'(\cos 2\theta \frac{\partial \theta}{\partial y} - \sin 2\theta \frac{\partial \theta}{\partial x}) = 0 \\
&\sin\varphi' \sin 2\theta \frac{\partial s}{\partial x} + (1-\sin\varphi' \cos 2\theta)\frac{\partial s}{\partial y} + 2s \sin\varphi'(\sin 2\theta \frac{\partial \theta}{\partial y} + \cos 2\theta \frac{\partial \theta}{\partial x}) = \gamma
\end{aligned} \tag{1.19}$$

These two partial differential equations can be shown to be of the hyperbolic type. A solution is obtained by considering the characteristic directions and obtaining equations for the stress variation along these characteristics (Atkinson and Potts (1975)). The differential equations of the stress characteristics are:

$$\frac{dy}{dx} = \tan[\theta - (\pi/4 - \varphi'/2)]$$
$$\frac{dy}{dx} = \tan[\theta + (\pi/4 - \varphi'/2)] \tag{1.20}$$

Along these characteristics the following equations hold:

$$ds - 2s\tan\varphi' \, d\theta = \gamma(dy - \tan\varphi' \, dx)$$
$$ds + 2s\tan\varphi' \, d\theta = \gamma(dy + \tan\varphi' \, dx) \tag{1.21}$$

Equations (1.20) and (1.21) provide four differential equations with four unknowns x, y, s, and θ which, in principle, can be solved mathematically. However, to date, it has only been possible to obtain analytical solutions for very simple problems and/or if the soil is assumed to be weightless, $\gamma=0$. Generally, they are solved numerically by adopting a finite difference approximation.

Solutions based on the above equations usually only provide a partial stress field which does not cover the whole soil mass, but is restricted to the zone of interest. In general, they are therefore not Lower bound solutions (see Section 1.9.3).

1.9.3 Limit analysis (upper bound and lower bound analysis)

- The upper/lower bound theorems of limit analysis are based on the following assumptions:
  (a) Soils behavior is **perfect/ideal plastic (理想塑性)**
  (b) Yield surface is **convex in shape (凸屈服面)** and plastic strains follow **an associative flow rule (normality)** (相关联的流动法则)
  (c) Changes in geometry of soil mass at failure are insignificant

- **Upper bound (unsafe) solution** 上限解
- **Lower bound (safe) solution** 下限解
- **The true solution in between**

Unsafe theorem **Upper bound：能量角度出发**

An unsafe solution to the true collapse loads (for the ideal plastic material) can be found by selecting any kinematically possible failure mechanism and performing an appropriate work rate calculation. The loads so determined are either on the unsafe side or equal to the true collapse loads.

This theorem is often referred to as the 'Upper bound' theorem. As equilibrium is not considered, there is an infinite number of solutions which can be found. The accuracy of the solution depends on how close the assumed failure mechanism is to the real one.

Safe theorem **Lower bound：应力状态角度出发**

If a statically admissible stress field covering the whole soil mass can be found, which nowhere violates the yield condition, then the loads in equilibrium with the stress field are on the safe side or equal to the true collapse loads.

This theorem is often referred to as the 'Lower bound' theorem. A statically admissible stress field consists of an equilibrium distribution of stress which balances the applied loads and body forces. As compatibility is not considered, there is an infinite number of solutions. The accuracy of the solution depends on how close the assumed stress field is to the real one.

Example: Critical height of a vertical cut in undrained clay

Unsafe solution (Upper bound)

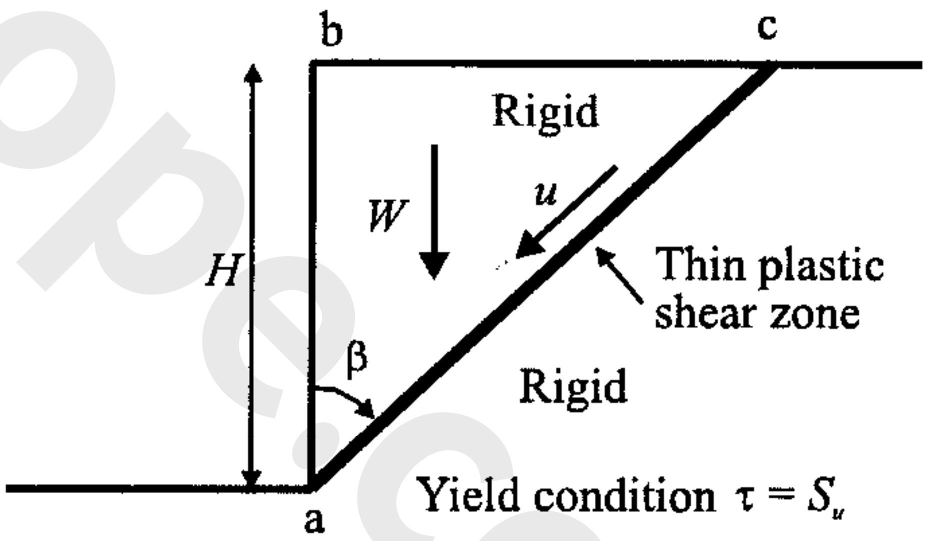

Figure 1.13: Failure mechanism for unsafe analysis

Rigid block 'abc' moves with respect to the rigid base along the thin plastic shear zone 'ac', Figure 1.13. The relative displacement between the two rigid blocks is u. Internal rate of energy dissipation is:

$$= u\, S_u\, H / \cos\beta \qquad (1.22)$$

Rate of work done by external forces is:

$$= \tfrac{1}{2} H^2 u \gamma \sin\beta \tag{1.23}$$

Equating equations (1.22) and (1.23) gives:

$$H = 4S_u/(\gamma \sin 2\beta) \tag{1.24}$$

Because this is an unsafe estimate, the value of β which produces the smallest value of H is required. Therefore:

$$\frac{\partial H}{\partial \beta} = -\frac{8S_u \cos 2\beta}{\gamma \sin^2 2\beta} \tag{1.25}$$

Equation (1.25) equals zero if $\cos 2\beta = 0$. Therefore $\beta = \pi/4$ which, when substituted in Equation (1.24), gives:

$$\boxed{H_{UB} = 4S_u / \gamma} \tag{1.26}$$

The same with LEM !

从另一方面说明LEM解答也偏危险

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Safe solution (Lower bound)

Stress discontinuities are assumed along lines *ab* and *ac* in Figure 1.14. From the Mohr's circles, see Figure 1.14, the stresses in regions 1 and 2 approach yield simultaneously as H is increased. As this is a safe solution, the maximum value of H is required. This occurs when the Mohr's circles for zones 1 and 2 just reach the yield condition. Therefore:

$$\boxed{H_{LB} = 2S_u / \gamma} \qquad (1.27)$$

$$\boxed{\gamma H = 2S_u}$$

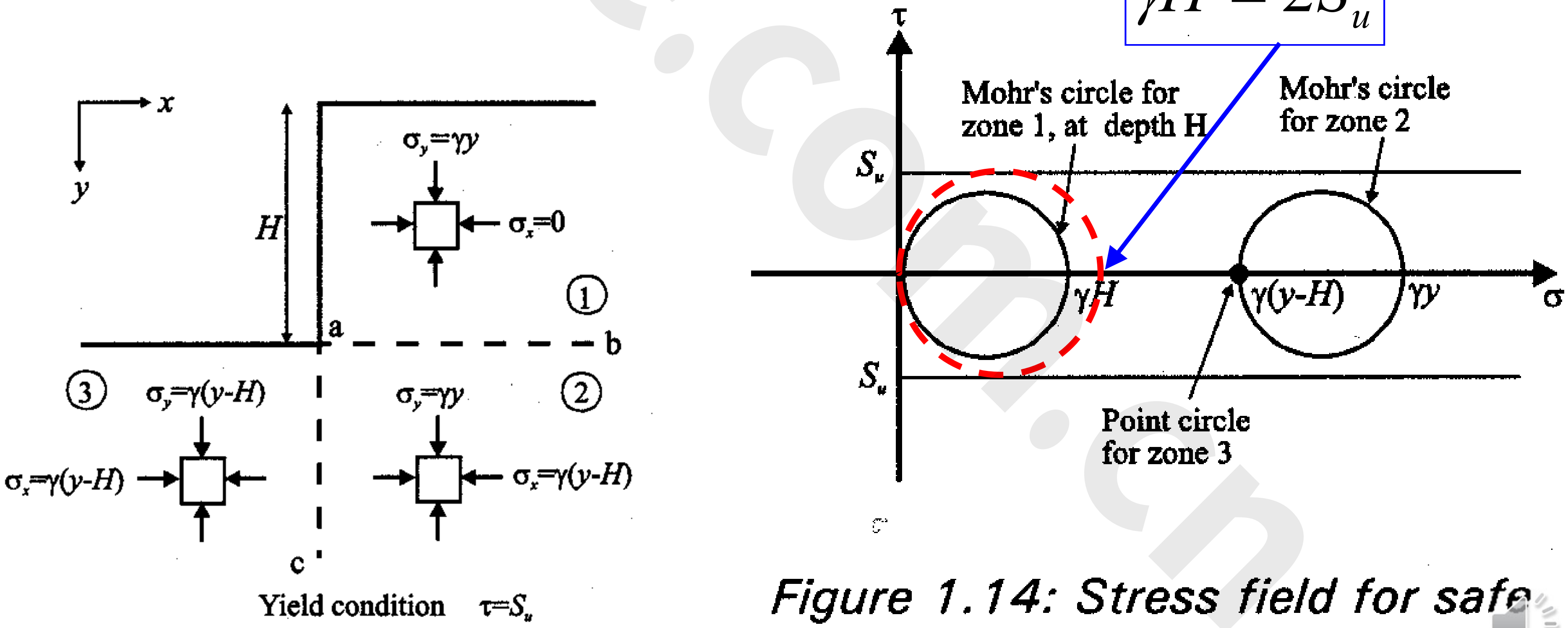
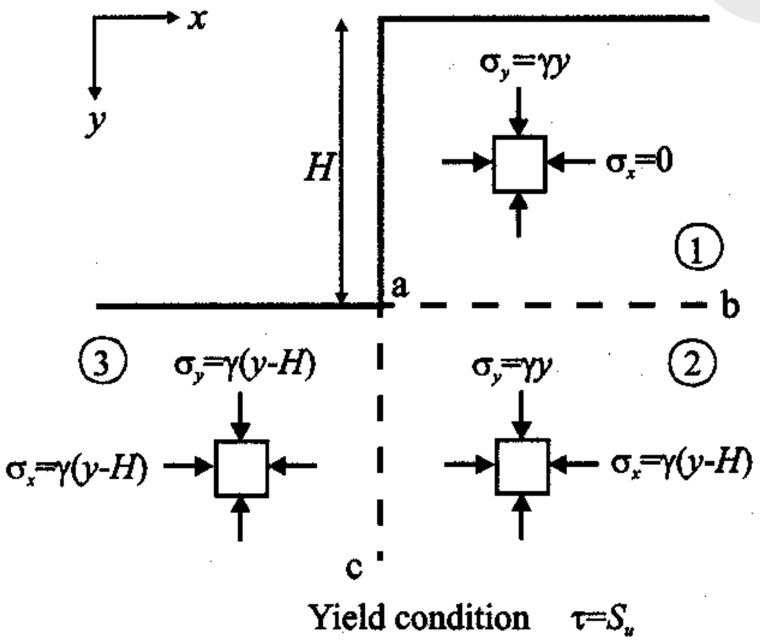


Figure 1.14: Stress field for safe solution

在这一问题上

极限平衡解答=上限解=2x下限解

从分析过程可以很明显地看出，下限解过于保守



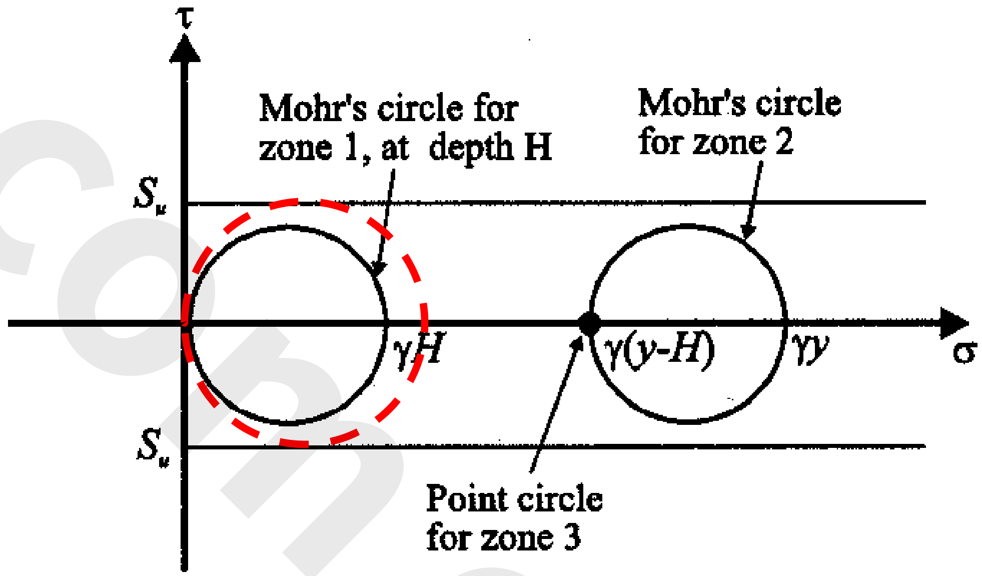


Figure 1.14: Stress field for safe solution

1.9.4 Comments

- None of the above methods satisfy all basic requirements.
- All methods are approximate.
- There are different solutions to the same problem.
- Solutions are for soil mass at failure only (limit).
- No displacements are obtained under working load.

- The methods are simple and provide useful solutions to stability problems (slopes, footings, etc.).
- A lot of experiences have been accumulated.

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1.10 Numerical analysis (数值分析)

1.10.1 Beam-spring approach (弹性地基梁法)

For soil-structural interaction:

- Soils are simplified as un-connected springs (linear or non-linear).
- Only a single structure is considered (a single pile or wall).
- Structural supports (props, anchors etc) are represented by springs.
- Limits (cut-offs) on the springs may be imposed. The limits are determined separately.
- Analytical and/or numerical solutions are obtained.
- Examples: Winkler's solution, FREW, etc

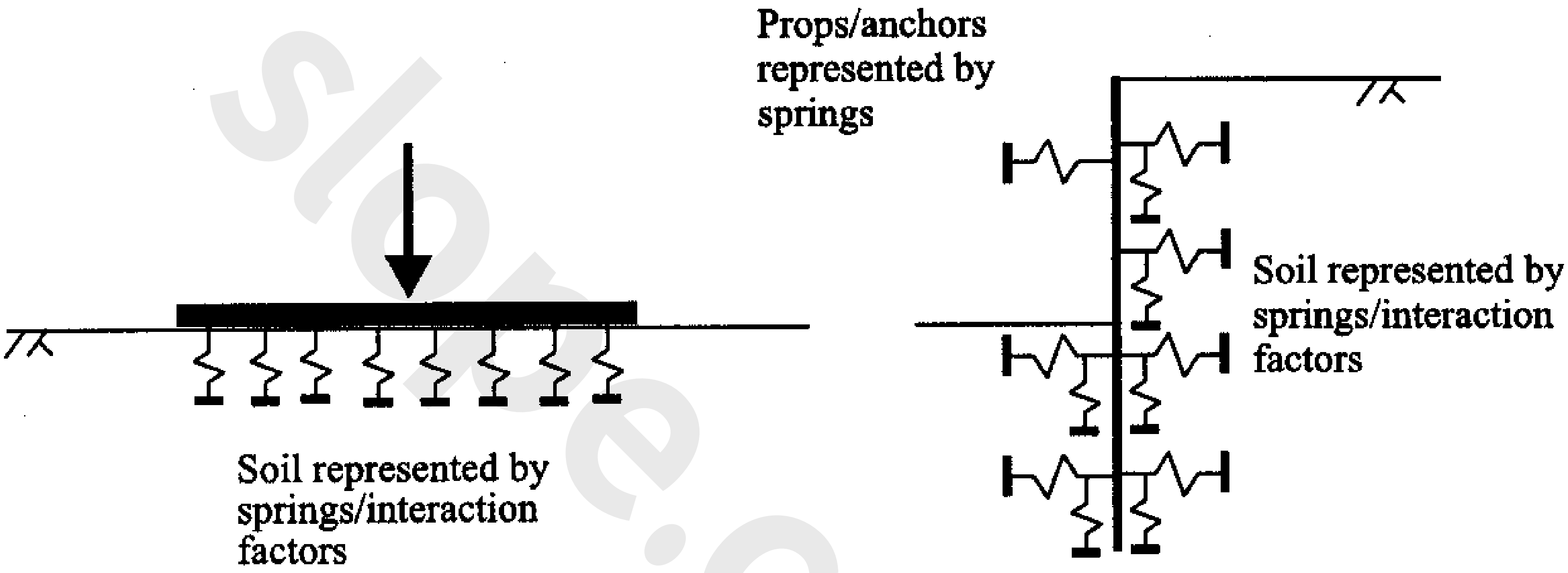

Figure 1.15: Examples of beam-spring problems

Winkler model 文科勒地基模型（基础工程课程中有详细讲解）
土体用一系列弹簧来模拟，而不是建立严密的本构关系

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1.10.2 Full numerical analysis (完全数值分析)

For full soil (+structural interaction) analysis:

- All basic requirements are satisfied.
- Limit loads and displacements are obtained.

- Examples: Finite element method (FEM), Finite difference methods (FDM), Boundary element method (BEM), and other methods;

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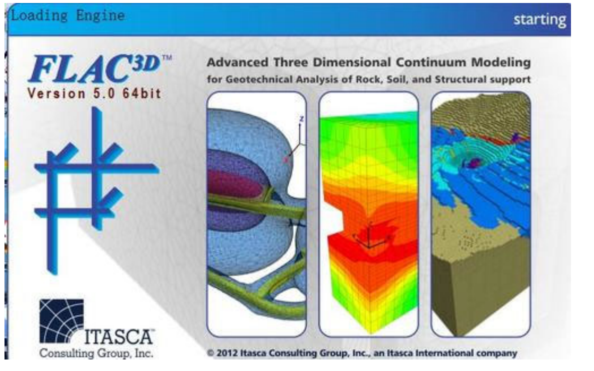
- Software:

FEM: **SIGMA/W**, **SEEP/W**, Plaxis, ABAQUS, ANSYS,…

FDM: FLAC^{2D} and FLAC^{3D}

MPM: Anura3D



One Model. One Tool. Many Analyses.

essential for geotechnical professionals

Anura3D
MPM Research Community
Modelling large deformation and soil–water–structure interaction

Table 1.1: Basic solution requirements satisfied by the various methods of analysis

	METHOD OF ANALYSIS		不同条件 SOLUTION REQUIREMENTS: Equilibrium	Compatibility	Constitutive behaviour	Boundary conditions: Force	Boundary conditions: Disp
1	Closed form		S	S	Linear elastic	S	S
2.1	Limit equilibrium		S	NS	Rigid with a failure criterion	S	NS
2.2	Stress field		S	NS	Rigid with a failure criterion	S	NS
2.3	Limit analysis	Lower bound	S	NS	Ideal plasticity with associated flow rule	S	NS
		Upper bound	NS	S		NS	S
3.1	Beam-Spring approaches		S	S	Soil modelled by springs or elastic interaction factors	S	S
3.2	Full Numerical analysis		S	S	Any	S	S

S - Satisfied; NS - Not Satisfied

Table 1.2: Design requirements satisfied by the various methods of analysis

	METHOD OF ANALYSIS	不同设计 DESIGN REQUIREMENTS						
		Stability			Wall & supports		Adjacent structures	
		Wall & support	Base heave	Overall	Structural force	Displacement	Structural force	Displacement
1	Closed form (Linear elastic)	No	No	No	Yes	Yes	Yes	Yes
2.1	Limit equilibrium	Yes	Separate calcul.	Separate calcul.	Yes	No	No	No
2.2	Stress field	Yes	Separate calcul.	Separate calcul.	Yes	No	No	No
2.3	Limit analysis: Lower bound	Yes	Separate calcul.	Separate calcul.	Crude estimate	No	No	No
	Limit analysis: Upper bound	Yes	Separate calcul.	Separate calcul.	Crude estimate	Crude estimate	No	No
3.1	Beam-Spring approaches	Yes	No	No	Yes	Yes	No	No
3.2	Full Numerical analysis	Yes	Yes	Yes	Yes	Yes	Yes	Yes

1.11 Summary

1. Geotechnical engineering plays a major role in the design of nearly all civil engineering structures.
2. Design of geotechnical structures should consider:
   - Stability: local and overall;
   - Structural forces: bending moments, axial and shear forces in structural members;
   - Movements of the geotechnical structure and adjacent ground;
   - Movements and structural forces induced in adjacent structures and/or services.
3. For a complete theoretical solution the following four conditions should be satisfied:
   - Equilibrium;
   - Compatibility;
   - Material constitutive behaviour;
   - Boundary conditions.
4. It is not possible to obtain closed form analytical solutions incorporating realistic constitutive models of soil behaviour which satisfy all four fundamental requirements.

5. The analytical solutions available (e.g. Limit equilibrium, Stress fields and Limit analysis) fail to satisfy at least one of the fundamental requirements. This explains why there is an abundance of different solutions in the literature for the same problem. These simple approaches also only give information on stability. They do not provide information on movements or structural forces under working load conditions.
6. Simple numerical methods, such as the beam-spring approach, can provide information on local stability and on wall movements and structural forces under working load conditions. They are therefore an improvement over the simpler analytical methods. However, they do not provide information on overall stability or on movements in the adjacent soil and the effects on adjacent structures or services.
7. Full numerical analysis can provide information on all design requirements. A single analysis can be used to simulate the complete construction history of the retaining structure. In many respects they provide the ultimate method of analysis, satisfying all the fundamental requirements. However, they require large amounts of computing resources and an experienced operator. They are becoming widely used for the analysis of geotechnical structures and this trend is likely to increase as the cost of computing continues to decrease.